Computational Thinking and Mathematical Problem Solving, an Analytics Based Learning Environment

White Paper #2

Erasmus+ KA220-SCH project CT&MathABLE: "Computational Thinking and Mathematical Problem Solving, an Analytics Based Learning Environment", #2022-1-LT01-KA220-SCH-000088736. https://www.fsf.vu.lt/en/ct-math-able

Executive Summary

The latest progress in the Erasmus+ project Computational Thinking and Mathematical Problem Solving, an Analytics Based Learning Environment (CT&MathABLE) provides comprehensive learning analytics driven support for developing Computational and Algebraic Thinking (CT and AT) in K-12 schools. Through the deployment of digital technology the project provides educators with new approaches to skills development that builds on well supported learning pathways and is individually tailored to the learner. This is achieved through a novel learning systems architecture which supports individualized development paths and seamless integration of CT and AT conceptual development with tailored problem solving and assessment frameworks. CT&MathABLE is about creating learning approaches more appropriate to students who have traditionally been disadvantaged, helping them to benefit from the integration of CT and AT. We present the curriculum analysis which forms the basis for learning pathways in AT in this briefing paper.

1 Introduction

As part of curricula reforms, many European countries have already included elements of Computational Thinking (CT) skills in compulsory schooling [Bocconi et al.(2022), Niemelä et al.(2022)]. CT can be understood as a way of thinking used to develop solutions in a form that ultimately allows executing those solutions. Thinking computationally means being able to approach and solve problems efficiently based on the principles and methods of computing [Arfé et al.(2020)]. CT is a type of analytical thinking that employs mathematical and engineering thinking to understand and solve complex problems within the constraints of the real world. This discussion is elaborated on by Denning and Tedre [Denning and Tedre(2021)], who also link the concept to many already pre-existing ideas of abstraction, decomposition, data representation and algorithms and their design. In developing our strategy for teaching Computer Science (CS), the following principles, proposed by [Dagiene et al.(2022)], are emphasized: 1) do not teach CS as an isolated subject, but teach CS as a part of science and technology, offering a deep contextual view; 2) teach the genesis of fundamental CS concepts and improve them step by step, 3) teach to control computers by programming and automate well-understood activities to make society more efficient. Algebraic Thinking is at the core of Mathematics and is an important component of the broader construct Mathematical Thinking. Algebraic thinking implies, among other things, acquiring the ability to represent and generalize patterns in any application area. We analyze their status and their relationship to the concepts taught in different courses, throughout Primary and Secondary Education. Although the development of Algebraic Thinking is possible in several subject contexts, we will focus on the subject of Mathematics. As CT shares some skills with Algebraic Thinking as a part of mathematics education, intertwining them should benefit the development of both [Bråting and Kilhamn(2021)]. Algebraic Thinking is defined as the ability to generalize, represent, justify, and reason with abstract mathematical structures and relationships. One of the most attractive ways to do this is to implement CT education through changes to school curricula, for example, to integrate in CS or other similar courses. For example, the Ministry of Culture and Education in Finland highlights new literacy competency, which includes ICT skills, media literacy and programming. In the national curriculum, programming is mentioned in the sections of cross-curriculum studies and subject studies (mathematics and craft) [**?**]. Activities and learning goals regarding programming are a part of the curriculum, whereas the assessment focuses on the subject knowledge. Lithuania on the other hand introduced compulsory CT education from early grades through to the end of compulsory schooling [Niemelä et al.(2022)]. To address this variation in curricula, an in depth analysis of 6 European national Mathematics curricula has been conducted with initial focus on students aged 9 to 14 years. This analysis forms the foundation for developing individualised learning pathways. Our analysis reveals considerable similarity, permitting development of core learning paths, however, there is considerable regional variation, which also requires specialisation modules in order to accommodate the six national curricula analysed so far.

2 Method

Establishing learning pathways of broad relevance requires an in-depth understanding of the relationships between conceptual development in the domain and linking this to conceptual progression within each topic or skill area. Our approach builds upon a review of related literature which establishes a research informed classification of AT skills and competencies. This classification structure was used to derive an initial set of codes with which we could annotate the curricula of six European countries (Finland, Hungary, Lithuania, Spain, Sweden, and Turkiye). Following this comprehensive analysis and classification of curricula a final coding structure was developed that captured the conceptual content of CT and AT visible in the curricula analysed.The procedure applied is summarised in Figure 1.

2.1 Definitions

We have defined the ability to think computationally as a combination of higher-order cognitive skills: (a) analytical thinking and decomposition (analyze a problem and break it into parts); (b) algorithmic thinking (plan and identify action sequences to get to its solution); (c) hypotheses testing and debugging (monitoring and correcting errors); and (d) abstraction. Likewise we have defined algebraic thinking as a multi-dimensional construct comprising abilities and awareness's connected to: categorization, classification, Problem solving, Comparison, sort,Counting or approximation, numerical representation, measurement and units, equations and operations, mental operations, mathematical objects and properties, spatial cognition, orientation in space or planes, data collection and manipulation, randomness, columnar operations, part-whole relationships, natural numbers and integers, sets, manipulating rationals and fractions, mathematical logic in everyday context, ratios and percentages, functions and relations, conceptualisation and manipulation of patterns, descriptive statistics, probabilities.

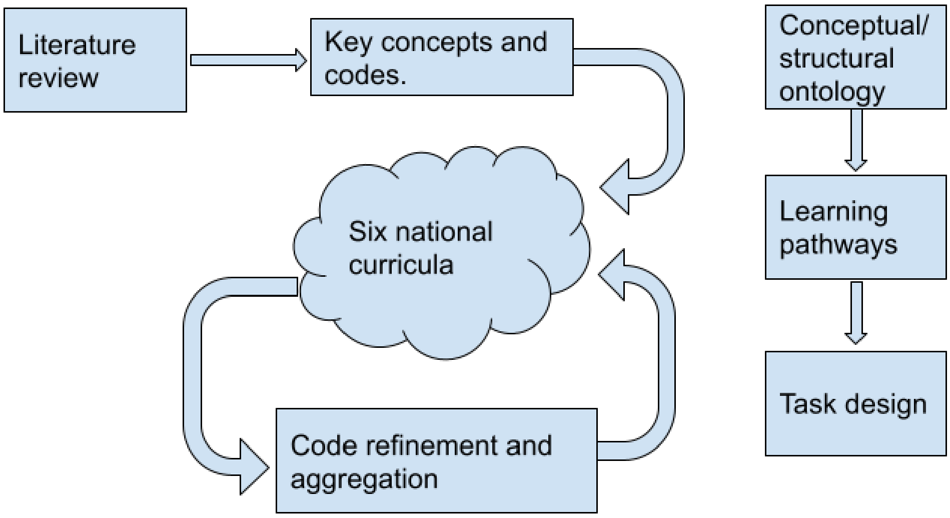

Figure 1: Analysis process

2.2 Analysis

As one might expect the six national curricula differed both structurally and in terms of content and order of introduction of concepts. A curriculum typically consists of a series of topics, and within each topic area a list of its detailed learning material and outcomes is specified. The curricula forming the empirical data for our study therefore need to be consolidated during analysis. Table 1 contains the number of detailed learning statements in each curriculum ordered by country. One reason for the richness of the Hungarian curriculum is that it contains two kinds of details. One is the preparation for the knowledge, and the other is the real learning outcome. Some topic details are divided into 2-3 parts in a country, while it is in only one row in the others. These differences were reduced during the steps of consolidation. Duplication, associated with cognitive progression in important topics was an important aspect of the analysis, since these sequences need to be incorporated into the CT&MathABLE learning pathways. After eliminating duplicates, the Hungarian curriculum was selected as a reference point, since it is the most detailed. Each row of the other curricula was assigned to the corresponding topic in the Hungarian curriculum, if it existed, or a new topic (and perhaps code) was created where needed.

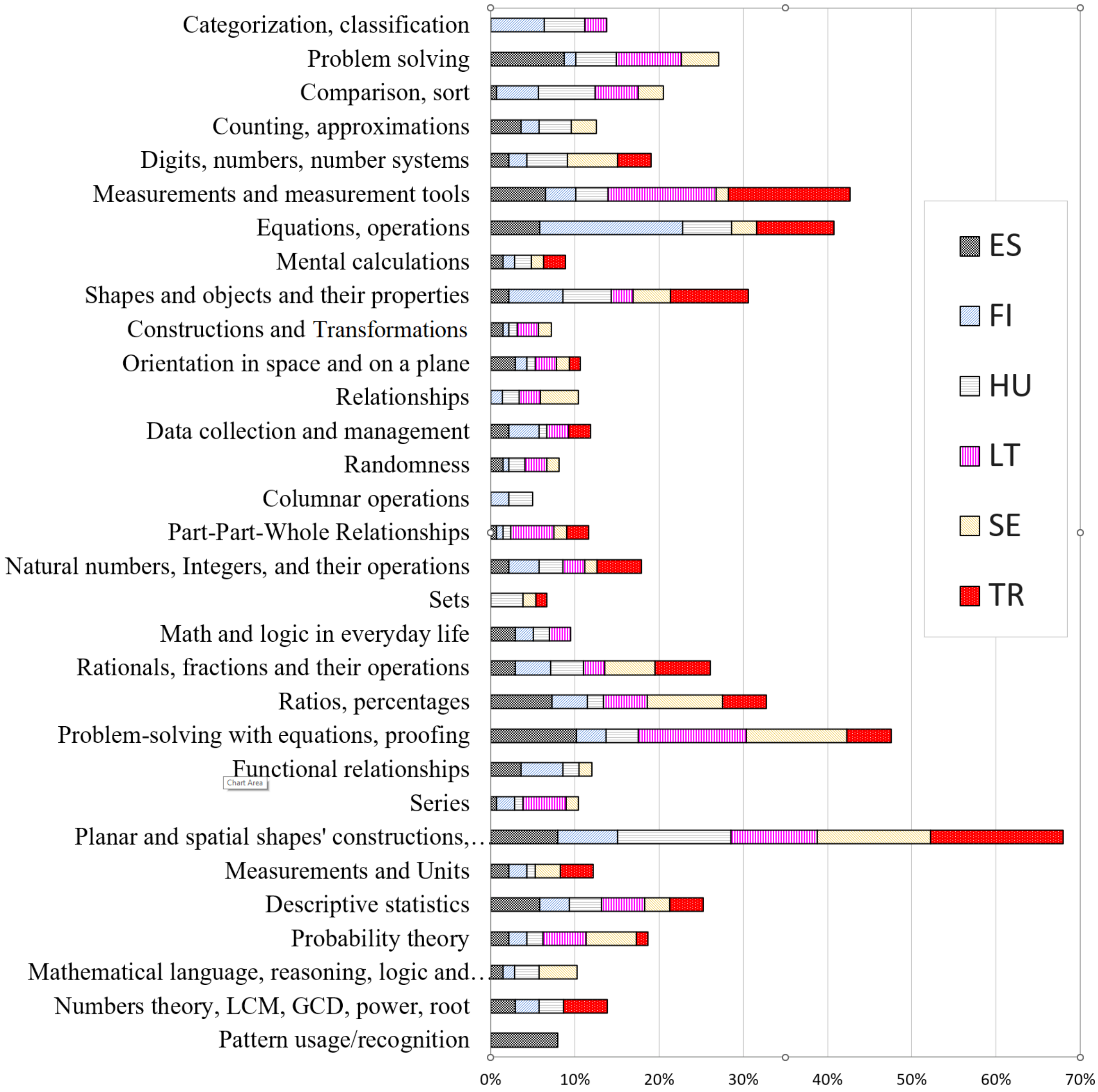

Figure 2: Curriculum comparison

3 Results

The most important outcome of the current study is the identification of topic areas and concentrations in the curricula of our sample of six countries. The countries we studied show strong correlation with commonality between curricula of over 47%, between any four countries and over 80% with another three. Each country has its own national focus which area with more coverage of that topic in comparison to the other countries studied. Spain places emphasis on problem solving and pattern recognition; in Finland equations and operations are most prevalent; for Hungary, comparison, sorting and equations; in Lithuania Measurements and Problem solving, Sweden has considerable focus on Problem Solving and Ratios; and for Turkey Measurements and Equations have great significance. A detailed comparison is found in Figure 2

4 Conclusions

We have analysed the mathematics education literature in detail in order to derive a definition of the cognitive development areas defined as Algebraic Thinking in the research literature. We have applied this definition, together with a higher order definition of Computational Thinking to the coding of statements in the mathematics curricula of six European nations. We find considerable similarity, but also interesting differences when we perform a statistical analysis of the frequency of reference to certain codes. Content analysis reinforces the claim that the core of the curricula is similar, close to 50% congruence. A general learning path is also observable, which starts from the simple objects with their classification and categorization, as they investigate their properties and relations. Based on the experience the learners recognize patterns, and they can generalize. As they learn the basic numerals, learners start to use symbols, either for numbers or for operations. Their mathematical vocabulary develops and extends, and finally these advanced integrative concepts and definitions combined with their arithmetical skills introduce them to problem-solving. This general learning path covers the major components of AT and can be used in the next stage of the project to develop the tasks needed to support individual learning trajectories within each path. This individualisation will be achieved by applying learning analytics to help each learner define an individual trajectory within the path, based on their prior task performance and demonstrated achievements.

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