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Learning Outcomes based on the curriculum analysis

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1. General overview

Creating a learning path involves several fundamental principles that guide students towards achieving specific learning objectives. This process entails designing a path with clear, SMART learning objectives, tailoring the path to each student's individual needs, incorporating various instructional methods, and providing ongoing assessment and feedback. By adhering to these principles, educators can develop effective learning paths that foster student engagement, motivation, and success.

This document consists of a detailed analysis of national curricula and is designed with clear, specific, measurable, achievable, relevant, and time-bound (SMART) learning objectives. These objectives help students understand what they are expected to achieve and how to accomplish their goals. Our learning paths are inherently pupil-centric, taking into account each student’s prior knowledge, interests, and learning styles based on input from the learning analytics system. This personalized approach ensures that students are presented with relevant tasks at an appropriate level, keeping them motivated and engaged in the learning process.

In response to curricula reforms, many European countries have incorporated Computational Thinking (CT) skills into compulsory schooling. CT is a way of thinking that involves developing solutions in a form executable by computers. It involves analytical thinking and employs mathematical and engineering principles to solve complex problems. Denning and Tedre (2021) link CT to pre-existing concepts like abstraction, decomposition, data representation, and algorithms.

Our strategy for teaching Computer Science (CS) emphasizes principles proposed by Dagiene et al. (2022): integrating CS within science and technology for contextual understanding, teaching fundamental CS concepts progressively, and using programming to automate well-understood activities for societal efficiency. Algebraic Thinking, a core component of Mathematical Thinking, involves representing and generalizing patterns and reasoning with abstract structures. Integrating CT with Algebraic Thinking in mathematics education can enhance the development of both.

Various European curricula reforms highlight the integration of CT skills. For instance, Finland's national curriculum includes ICT skills, media literacy, and programming across subjects, while Lithuania mandates compulsory CT education from early grades. Our analysis of six European national mathematics curricula (Finland, Hungary, Lithuania, Spain, Sweden, and Türkiye) forms the foundation for individualized learning pathways. Despite regional variations, core learning paths can be developed, supplemented by specialized modules to accommodate specific national curricula.

2. Method

Our approach involves reviewing related literature to create a research-informed classification of Algebraic Thinking (AT) skills and competencies. This classification was used to annotate the curricula of six European countries. Following a comprehensive analysis and classification, a final coding structure was developed to capture the conceptual content of CT and AT visible in the analyzed curricula. The analysis process is summarized in Table 1.

2.1 Definitions

We define computational thinking as a combination of higher-order cognitive skills: (a) analytical thinking and decomposition (analyzing a problem and breaking it into parts); (b) algorithmic thinking (planning and identifying action sequences to solve it); (c) hypothesis testing and debugging (monitoring and correcting errors); and (d) abstraction. Similarly, algebraic thinking is a multi-dimensional construct encompassing abilities and awareness connected to categorization, classification, problem-solving, comparison, sorting, counting or approximation, numerical representation, measurement, equations and operations, mental operations, spatial cognition, data collection, randomness, part-whole relationships, natural numbers, sets, rational numbers, mathematical logic, ratios, functions, patterns, statistics, and probabilities.

Table 1. Algebraic Thinking categories and their description

AT	***AT categories by NCTM***	***Detailed description of AT categories***
Equality	Studying and representing relationships	Equality and the concept of equivalence (Students often have the misconception that the equal sign means “the answer is” when it really means “the same as.” True/false equations are a way to expose students to the meaning of equality, such as 5 + 2 = 3 + 4, or 8 = 2 + 6.)
Inequality		Inequality (Develop in your students a conceptual understanding of greater than and less than as relational symbols, and not rely on memory tricks.)
Relational thinking		Relational thinking (Relational thinking focuses on the why behind the right answer. For example, 5 x 3 = 15, but why? It is because there are three groups and each group has five.)
Problem	recognizing and analyzing	Problem solving and critical thinking (Students who have problem-solving and critical thinking skills can solve problems in new

solving	patterns	contexts and can generalize to new situations.)
Patterns		Patterns (“We need to train children to look for, and to expect to find, patterns in all math work that they do”)
Generalization	making generalizations	Making generalizations (It is important that students discover patterns, which include mathematical rules, in order to make conjectures about the growing pattern.)
Variables / Unknowns	analyzing how things change	Variables (Variables are unknown and can change, and are represented by symbols. Teachers should explicitly explain to students that the one-letter symbol is an abbreviation)
Numbers		Positive and negative numbers (Students should be exposed to some negative numbers in the early grades. When teachers say, “You can’t take 6 from 3,” or “You can’t subtract a small number minus a big number,” teachers are giving students information that just isn’t true.)
Symbols		Symbolic representation of mathematical ideas (Learning that equations communicate the relationship between numbers is crucial for a conceptual understanding of the symbols.)

2.2 Analysis

As expected, the six national curricula varied in structure, content, and the order of introducing concepts. Each curriculum consists of a series of topics, with detailed learning materials and outcomes specified within each topic area. For our analysis, we needed to consolidate these curricula. Table 1 lists the number of detailed learning statements in each curriculum by country. The Hungarian curriculum is particularly rich in detail, containing both preparatory knowledge and actual learning outcomes. Some topics are divided into 2-3 parts in one country, while presented in a single row in others. These differences were minimized during consolidation. Duplication related to cognitive progression in important topics was crucial, as these sequences need to be incorporated into the CT&MathABLE learning pathways.

After eliminating duplicates, the Hungarian curriculum was used as a reference point due to its detailed nature. Each row of the other curricula was matched to the corresponding topic in the Hungarian curriculum or a new topic (and possibly code) was created if necessary.

3. Analytical Results

The key outcome of this study is the identification of topic areas and concentrations within the curricula of our six-country sample. The analysis revealed a strong correlation, with over 47% commonality between any four countries and over 80% between three countries. Each country also has its unique national focus, with more extensive coverage of certain topics compared to others. Spain emphasizes problem-solving and pattern recognition; Finland focuses on equations and operations; Hungary highlights comparison, sorting, and equations; Lithuania prioritizes measurements and problem-solving; Sweden concentrates on problem-solving and ratios; and Turkey gives significant importance to measurements and equations. A detailed comparison is shown in Tables 2, 3, and 4.

Table 2. Detailed description of mathematical topics for age groups 9-10

	Age group 9-10 (grade 3-4)		
Math Topic		**Topic categories based on curriculum analysis**	**Short description of outcomes in LP based on the outcomes of the analysis**
Geometry	9	Shapes and objects, and their properties	freely or from an example builds and constructs shapes and patterns from given objects, two-dimensional shapes; recognizes and is able to continue a line pattern or planar pattern
			constructs bodies from edges and faces; prepares edge frames and nets of objects; finds all fitting multiple criteria; symmetry
			Basic two- and three-dimensional geometrical objects and their properties and relationships. Construction of geometrical objects, both with and without digital tools.
			Realize that shapes such as squares and rectangles have more than one line of symmetry. Complete the given figure according to the horizontal or vertical line of symmetry, Covering, drawing the covering pattern on dotted or squared paper. Express more abstract concepts such as point, line, ray, line segment, and give examples from their surroundings by recognizing the angle.
			Realize that shapes such as squares and rectangles have more than one line of symmetry, and complete the given figure according to horizontal or vertical lines of symmetry. Allow overlay by drawing the overlay pattern on dotted or plaid paper.
			Find a certain geometric pattern through experience. Determine the relationship in a pattern whose elements are objects, geometric shapes, or other objects, and finding the missing element is included. Create geometric patterns with at most three elements.

			Simple two-dimensional geometric figures in everyday objects: identification and classification according to their elements. Basic geometric vocabulary: verbal description of the elements and properties of simple geometric figures. Properties of two-dimensional geometric figures: exploration using manipulative materials and digital tools.
			Geometric figures of two or three dimensions in everyday objects: identification and classification according to their elements and the relationships between them. Vocabulary: verbal description of the elements and properties of simple geometric figures.
			Build, draw, examine, and classify shapes. Classify shapes into cylinders, cones, and other shapes.
			The students are guided to identify and name the qualities of shapes, and they classify shapes using these.
			Classify 2D shapes into polygons and others, and study their properties.
			Students develop their ability to visualize a three-dimensional environment and observe plane geometry in it.
			Study symmetry about a line.
			Learn about the concepts of point, segment, line, and angle.
			The students are guided to identify and name the qualities of shapes, and they classify shapes using these.
			differentiates between objects and two-dimensional figures;
			identifies, tells apart, and describes objects, things, and people based on properties
			names the properties, curves, shapes, and straight lines; recognizes reflexively symmetrical and not reflexively symmetrical shapes
			rectangle, square, rectangular cuboid, cube properties and differences
			Learning to describe the position of these shapes relative to each other
			Basic geometrical two-dimensional objects, spheres, cones, cylinders, and cuboids. Properties of these objects and their relationships. Construction of geometric objects.
			The symmetry in everyday life and how symmetry can be constructed.
			Classify shapes according to the number of corners and sides, to name triangles, squares, rectangles, and circles, to recognize them, and to create models. Circle. Other shapes are expected to be classified according to the number of sides and corners. Create, draw shape models using a single known shape and different shapes. Recognize and model geometric objects.
			The faces, corners, and edges. Determine the similarities and differences between cube, square, and rectangular prisms.

			Using a ruler to draw triangles, squares, and rectangles, determine the diagonals of the square and rectangle.
			There are naming the sides and corners of triangles, squares, and rectangles, determining the side properties, and classifying the triangles according to their side lengths. Create structures suitable for drawings created with isometric or squared paper and identical cubes.
	11	Orientation in space and on a plane	Description of the relative position of objects in space or their representations, using appropriate geometric vocabulary (parallel, perpendicular, oblique, right, left, etc.). Verbal description and interpretation of movements, in relation to oneself or to other points of reference, using appropriate geometric vocabulary. Interpretation of itineraries in plans, using physical and virtual supports.
			Geometric models in solving problems related to the other senses. Geometric relations: recognition in the environment.
			Geometric models in addressing issues and finding solutions related to the other senses. Recognition of geometric relationships in fields outside the mathematics class, such as art, science, and everyday life.
			Students practice using the concepts of direction and location.
			Guide the students to use the scale when using the map.
			properly uses terms describing directions and distances in two- and three-dimensions; is able to navigate their neighborhood and on a map
	25	Planar and spatial shapes' constructions, transformations, properties, and classification	Measure and calculate the perimeters and areas of different shapes and the volumes of rectangular cuboids.
			Calculate the perimeters and areas of polygons.
			Learn more about triangles, quadrilaterals, and circles.
			groups triangles based on their angles and sides;
			knows the properties of quadrilaterals: the sum of interior and exterior angles, differences between convex and concave, the concept of a diagonal;
			knows the special quadrilaterals: trapezoid, parallelogram, rectangle, kite, rhombus, isosceles trapezoid, square; set diagram based on their properties

			uses the special properties of triangles and special quadrilaterals to solve problems; calculates circumference and area
			knows and uses the Pythagorean theorem to solve problems; knows the parts of circles, differentiates between line, half-line, and section
			Break a flat shape into parts. Join several shapes into a shape. Identify the missing parts of a shape or puzzle.
			Methods for determining and estimating the circumference and areas of different two-dimensional geometrical figures.
			Explain, show, and draw basic geometric concepts such as line, line segment, and ray. Name polygons and recognize their basic elements of rectangle, parallelogram, rhombus, and trapezoid.
			Calculate the length of the circle and the segment of the circle, and the area of the circle and the circle slice, by evaluating the arcs in which the central angle is seen in the circle.
			Understand the Pythagorean relation and solve the related problems. Translation and reflection transformations.
			Polygons, regular polygons, and their interior and exterior angles, solve the related area problems by examining rectangles, parallelograms, trapezoids, and rhombuses, and creating area relations of trapezoids and rhombuses.
			Calculate the perimeters of polygons, and the areas of rectangles in square centimeters and square meters. Recognize rectangular prisms, determine their basic properties, draw surface expansions, and calculate surface areas.
			Construction techniques of geometric figures by composition and decomposition, using manipulative materials, drawing instruments, and computer applications.
			Construction of geometric figures with manipulative and digital tools (dynamic geometry programs, augmented reality...).
			Location and movements on plans and maps from reference points (including cardinal points), directions, and calculation of distances (scales): description and interpretation with the appropriate vocabulary in physical and virtual supports.
			Transformations by means of rotations, translations, and symmetries in everyday life situations: identification of transformed figures, generation from initial patterns, and prediction of the result. Similarity in everyday life situations: identification of similar figures, generation from initial patterns, and prediction of the result.
			Elementary transformations such as rotations, translations, and symmetries in various situations using technological or manipulative tools.

			Strategies for calculating areas and perimeters of plane figures in everyday life situations. Geometric models in solving problems related to the other senses. Elaboration of conjectures about geometric properties, using drawing instruments (compass and protractor) and dynamic geometry programs. Geometric ideas and relationships in art, science, and everyday life.
			Geometric modeling: numerical and algebraic relationships in problem solving. Geometric relations in mathematical and non-mathematical contexts (art, science, daily life...).
			Practice geometric construction.
			Learn about the concept of scale and use it in enlargements and reductions.
			Strengthen the understanding of the concepts of similarity and congruence.
			gains practical experience in movement along a plane;
			knows the basic constructions: creating a perpendicular bisector, angle bisector, parallel and perpendicular lines, and copying an angle.
			constructs the reflexive and point mirror image of shapes; accurately constructs an image meeting multiple criteria;
			recognizes reduction and enlargement in everyday situations;
			knows and uses dynamic geometric software, understands the opportunities of applications.
			Recognize horizontal or vertical translations of an object by the number of cells. Recognize the rotation of an object around a point.
			Learn to prove the congruency of shapes in the coordinate plane by showing the sequence of transformations to get from one shape to another.
			Learn how to formulate the converse of a conditional statement. Through case studies, it is shown that not every inverse statement is true. A variety of mathematical and practical problems are addressed. Problems involving the combination of existing knowledge of shapes with knowledge of other areas
			Geometrical objects and their properties and relationships. Construction of geometrical objects, both with and without digital tools.
			Geometrical theorems, formulae, and arguments for their validity.
			Mathematical similarities and how the equal sign is used to draw equations and functions.
			Scale for reduction and enlargement of two- and three-dimensional objects.
			Uniformity and congruence.

			Scale for enlargement and reduction, and the use of scale in student-centered situations.
			Symmetry in the plane and how symmetry can be constructed.
			The concepts of bisector, corresponding, inverse, interior reverse, and exterior reverse angles are discussed, and their properties are examined.
			The concepts of congruence and similarity in polygons are examined, and congruent and similar polygons are identified and constructed. Vertical prism, right cylinder, right pyramid, and cone are geometric objects.
			Spatial relationships: location and description by means of geometric coordinates and other representation systems.
			Study 3D shapes. Learn to calculate the areas and volumes of spheres, cylinders, and cones.
			Students take a closer look at a rectangular cone, a circular cylinder, a circular cone, and a pyramid.
			creates the net of a cube, a rectangular cuboid, a prism, and a pyramid; creates objects based on pictures, views, plans, and nets;
			knows the following properties of the cube, rectangular cuboid, prism, and pyramid: types, counts, and relative position of faces; number of edges and vertices, face diagonal, space diagonal;
			determines the surface area and volume of right prisms, rectangular cuboids, or cube-shaped objects through measurements, calculates the surface area and volume of right prisms using a formula, and understands the relationships underlying the equation.
			knows the properties of the sphere; utilizes the properties of the cube, rectangular cuboid, prism, and pyramid to solve problems.
			Comparing, estimating, and measuring length, area, mass, volume, time, and angles using common units of measurement, including unit conversions related to them.
Analysis	10	Constructions and Translations	Strategies and techniques for the construction of two-dimensional geometric figures by composition and decomposition, using manipulative materials, drawing instruments (ruler/square), and computer applications. Properties of two- and three-dimensional geometric figures: exploration through manipulative materials (grids, geoplanes and polycubes) and the use of digital tools (dynamic geometry programs, augmented reality, educational robotics).
			Identification of transformed figures by means of translations and symmetries in everyday life situations. Generation of transformed figures from symmetries and translations of an initial pattern and prediction of the result.

			Guide the students to observe rotational and translational symmetry in their surroundings, for example, in art.
			creates a mirror image with movement, masking; creates symmetrical shapes; checks the correctness of a reflection, symmetrical line pattern
	23	Functional relationships	Learn about the first quadrant of the coordinate system and extend then to all quadrants.
			Coordinate system and grading of coordinate axes.
Measuring	6	Measurements and measurement tools	Comparison and ordering strategies of measures of the same magnitude (km, m, cm, mm; kg, g; l and ml): application of equivalences between units in everyday problems that involve converting to smaller units. Estimation of measures of length, mass, and capacity by comparison. Evaluation of the results of measurements, estimations, or calculations of measurements.
			Measurable attributes of objects (length, mass, capacity, surface area, volume, and angle amplitude). Conventional (km, m, cm, mm; kg, g; l and ml) and non-conventional units in everyday situations. Measurement of time (year, month, week, day, hour, and minutes) and determination of the duration of time periods.
			Strategies for measuring with non-conventional (repeating a unit, use of grids, and manipulative materials) and conventional instruments and units. Measurement processes using conventional instruments (ruler, tape measure, scales, analog and digital clocks).
			Calculation and estimation of amounts and change (euros and cents) in everyday life problems: income, expenses, and savings. Responsible purchasing decisions.
			Strategies for calculating the perimeters of plane figures and their use in solving everyday problems.
			Practice measuring and pay attention to accuracy, result evaluation, and checking the measurement.
			knows the standard units of time measurement: hour, minute, second, day, week, month, year;
			knows local and foreign currency denominations for numbers up to 10,000; knows how to exchange currency among different denominations
			measures the area and circumference of different polygons (possibly by coverage);
			approximates and measures using known units of measurement; knows the real-world size of units of measurement, converts among units of measurement

			Explain the relationship between the perimeters of the square and the rectangle and the side lengths. Realize that the areas of the shapes are composed of unit squares covering these areas. On the other hand, there are gains in associating the area of a square and a rectangle with multiplication and addition operations.
			Real-world decision problems using calculations (earnings, expenses, donations, savings...)
			Upscale or downscale units of time measurement. Concepts of path and speed. Relationship between path, time, and speed.
			Measuring area and circumference, length, our money, measuring time, weighing, and liquid. Know the millimeter and its relationship with other measurement units.
			Explain what the standard liquid measuring unit is and its necessity, and how to make measurements with liters and half liters. Measure by using liters and milliliters together, and to estimate the amount of liquid in a container with the measurement units they have learned.
			Read the time in minutes and hours. Explain the relationship between year-week, year-day, minute-second without conversion operations. Hour-minute, minute-second, year-week, year-month-week-day relations and expressing one in terms of the other are discussed.
			Notice and compare the relationship between money and coins, show this relationship, and solve the problems related to these relationships.
			Recognition of where kilograms and grams are used and explanation of the relationship between these units. Estimate and investigate the accuracy after estimating the masses of the objects. Half and quarter kilograms are measured in grams, and kilograms and grams are used together when measuring mass. Achievements of estimating the places where tons
	26	Measurements and Units	Practice drawing angles, measuring, and classifying.
			Guide the students to understand how the system of measurement units is structured. Practice unit conversions with the most commonly used measurement units.
Algebra	2	Problem solving	Compare different strategies to solve a problem in a patterned way. Obtain possible solutions to a problem following a known strategy. Demonstrate the mathematical correctness of the solutions to a problem and their coherence in the given context.
			A guided process of solving everyday problems.

			A structured process of modelling using mathematical representations (graphs, tables, etc.) to facilitate the understanding and resolution of everyday problems.
			Searches for a solution for problems; recalls their memory, re-states, and solves problems
			interprets and checks the solution; asks questions based on the problem, and illustrates the problem
			Solve multi-step problems; Up- or downscaling of units of measurement might be needed
			Analyze simple mathematical conjectures by investigating patterns, properties, and relationships in a patterned way. Give examples of problems about everyday situations that are solved mathematically.
			Interpret, verbally or graphically, problems of daily life, understanding the questions posed through different strategies or tools, including technological ones. Produce mathematical representations through schemes or diagrams that help in the resolution of a problematized situation.
			Make connections between different mathematical elements, applying their own knowledge and experiences. Interpret situations in diverse contexts, recognizing connections between mathematics and everyday life.
			Recognize simple mathematical language present in everyday life in different formats, acquire basic specific vocabulary, and show understanding of the message. Explain mathematical processes and ideas, the steps followed in solving a problem, or the results obtained, using simple mathematical language in different formats.
			Get acquainted with the concept of the unknown (variable)
			distinguishes between known and to be determined (unknown) data, and between relevant and irrelevant data
			utilizes a mathematical model, checks the results, and composes an answer
			Learn through examples: equation, unknown of an equation, solution of an equation. Use alternate equations to describe the same problem.
			Calculate the value of an expression with a letter in it, given the value of a letter. Form a letter expression from a word problem.
			Formulation of mathematical questions based on everyday situations.
			Unknown numbers and how they can be represented by a symbol.
	3	Comparison, sorting	Sorts based on own criteria, recognizes criteria in existing sorting, able to continue sorting
			compares finite sets based on the number of elements; pairs up elements of two sets with each other (1:1)

			understands and properly uses the more, less, equal amount relations, and the smaller, greater, equal relations with regard to numbers
			Mathematical similarities and how the equals sign is used to draw simple equations.
			orders numbers and quantities by size; gives and understands numbers constructed through various operations;
			able to find the position of numbers on number lines and tables, recognizes numbers in their different forms up to 10,000
			Students deepen their skills in comparing, classifying, and ordering, searching for answer options systematically, and observing cause and effect relationships in maths.
			sorts elements, inserts new elements
			Students deepen their skills in comparing, classifying, and ordering, searching for answer options systematically, and observing cause and effect relationships in maths.
	4	Counting, approximations	Strategies and techniques for interpreting and manipulating the order of magnitude of numbers (tens, hundreds, and thousands). Reasoned estimates and approximations of quantities in problem-solving contexts.
			counts and measures using (arbitrarily chosen or standard) units for numbers up to 10,000; can count up and down by tens, hundreds, thousands;
			knows the following approximation methods: approximate counting, approximate measurement, measuring with a multiple of the measurement unit; knows how to refine their approximation by re-approximation.
			Varied counting strategies, systematic counting, and adapting counting to the size of numbers in everyday situations in quantities up to 9999.
			approximation, evaluation of approximation
			Assessing plausibility when using estimates and calculations.
			uses tools (abacus), understands multiplication and division by 10, 100, 1000
			Assessing plausibility when using estimates and calculations.
	7	Equations, operations	Equality is an expression of an equivalence relationship between two elements and obtaining simple unknowns (represented by a symbol) in either element.
			Representation of the relationship " greater than" and "less than", and use of the signs < and >.

			Addition, subtraction, multiplication, and division of natural numbers are solved with flexibility and sense in contextualized situations: solving strategies, tools, and properties.
			Natural numbers and fractions in everyday life contexts: comparison and ordering. Relationships between addition and subtraction, and multiplication and division: application in everyday contexts.
			Equality and inequality relations, and use of = and ? signs between expressions involving operations and their properties.
			The understanding of the structure, connections, and divisibility of numbers is diversified by studying and classifying numbers.
			Ensure that the students master the concept of multiplication. Learn multiplication tables 6-9. Ensure that the students master the multiplication tables 1-10.
			Practice dividing into parts. (38/4=36/4+2/4)
			Ensure that the students master the concept of multiplication. Learn multiplication tables 6-9. Ensure that the students master the multiplication tables 1-10.
			Ensure that the students master the concept of multiplication. Learn multiplication tables 6-9. Ensure that the students master the multiplication tables 1-10.
			Practice both partitive and quotative division.
			Practice both partitive and quotative division.
			properly interprets the addition, subtraction, multiplication, and division for numbers up to 10,000;
			properly utilizes operations for a word problem, is able to use inverse operations
			understands the following terms: addend, sum, minuend, subtrahend, difference, multiplicand, multiplier, product, dividend, divisor, quotient, remainder
			knows the symbols for operations, uses parentheses for multiple operations
			How natural numbers and simple numbers as fractions are used in student-centered situations.
			Properties of the four operations, their relationships, and their use in different situations.
			Reading and writing 4, 5, and 6-digit numbers, dividing them into parts, and specifying the place values.
			Reading and writing 4, 5, and 6-digit numbers, dividing them into parts, and specifying the place values.

	8	Mental calculations	Mental calculation strategies with natural numbers and fractions. Strategies for recognizing which simple operations (addition, subtraction, multiplication, division as division and partition) are useful to solve contextualized situations. Construction of the multiplication tables based on the number of times, repeated addition, or grid arrangement.
			Practice the basic operations, mental calculation (+,-,*,/).
			accurately adds and subtracts in the head for numbers up to 100, multiplies and divides
			understands the relationship of multiplication and division tables;
			Methods of calculating using natural numbers when calculating mental arithmetic and approximate estimates, and written calculation. Use of digital tools in calculations.
			Reinforcing the mental multiplication and division operations.
			Reinforcing the mental multiplication and division operations.
	12	Relationships	Students deepen their skills in comparing, classifying, and ordering, searching for answer options systematically, and observing cause and effect relationships in maths.
			Students deepen their skills in comparing, classifying, and ordering, searching for answer options systematically, and observing cause and effect relationships in maths.
			finds pairs in a memory game; recognizes and expresses relationships; looks for patterns among elements of a series
			creates a series based on a given rule; lists months; recognizes relationships among element doubles or triples
			Extends, describes sequences of 2-4 repeating members. Differences can be in size, color, line thickness, angle of rotation, and sequence might carry over to the next line. Investigate sequences obtained by merging two sequences.
			Simple patterns in number sequences and simple geometric patterns, and how they are constructed, described, and expressed.
			Patterns in number sequences and geometrical patterns, and how they are constructed, described, and expressed.
			Simple tables and diagrams, and how they are used to categorize data and describe results from investigations, both with and without digital tools.

	15	Columnar operations	Practice addition and subtraction algorithms, ensuring that the skill is learned. (columnar addition and subtraction)
			Practice addition and subtraction algorithms, ensuring that the skill is learned. (columnar addition and subtraction)
			properly executes addition and subtraction in writing;
			Practice the multiplication algorithm and ensure that the skill is mastered (columnar multiplication).
			properly executes multiplication in writing with one- and two-digit multipliers, and division with one-digit divisors; approximates
			interprets and checks their solution.
	16	Part-Part-Whole Relationships	Proper fractions with a denominator up to 12 in everyday contexts.
			Learn the concept of fractions and practice basic calculations of fractions in different situations.
			illustrates, draws, measures, and understands the unit fractions and their multiples
			Compare fractions m/n < 1 where numerators or denominators are the same. Add and subtract decimal numbers with 1 or 2 decimal places
			Fractions as part of a whole and part of a number, and how parts are named and expressed as simple fractions. How simple fractions are related to natural numbers.
			Division (grouping, segmentation) process, fractionally introduced by emphasizing the part-whole relationship. The concept of a unit fraction and the relationship between the numerator and the denominator is reinforced.
			Define and use simple, compound, and integer fractions, and the addition and subtraction of fractions. Add and subtract fractions with equal denominators and solve appropriate problems.
	17	Natural numbers, Integers, and their operations	Ground the concept of a negative number and expand the number range with negative integers.
			directed units (temperature); understands lesser and greater relations for negative numbers too
			Students are guided to round numbers and calculate with approximations so that they learn to estimate the order of magnitude of the result.

			Students are guided to round numbers and calculate with approximations so that they learn to estimate the order of magnitude of the result.
			The four basic arithmetic operations (addition, subtraction, multiplication, and division) and rules for their use in calculations with natural numbers.
	20	Rational numbers, fractions, and their operations	Students familiarize themselves with decimal numbers as part of the decimal system and practice basic calculations with decimal numbers.
			Rational numbers, including negative numbers, and their properties, and how the numbers can be divided and used.
			The positional number system and how it is used to describe whole numbers and decimal numbers.
			Learn the concept of fractions and practice basic calculations of fractions in different situations.
			Students familiarize themselves with decimal numbers as part of the decimal system and practice basic calculations with decimal numbers.
			How numbers in fractions and decimals can be used in everyday situations.
	22	Problem-solving with equations, proofing	Study equations and find solutions by reasoning and experimenting.
			Study equations and find solutions by reasoning and experimenting.
			Formulation of mathematical questions based on everyday situations.
			Strategies for solving mathematical problems in student-related situations.
			Methods, including algebraic, for solving simple equations.
			Variables and their use in simple algebraic expressions and equations.
	24	Series (Sequences)	relations between decimal numbers (3-4 digits)
			Studying the regularity of number sequences and continuing number sequences according to a rule.
	30	Number theory, LCM, GCD, power, root	place-value in the decimal numeral system

	31	Pattern usage/recognition	Verbal description from regularities in a collection of numbers, figures, or pictures.
			Identification, verbal description, representation, and reasoned prediction of terms from regularities in a collection of numbers, figures, or pictures.
			Reasoned representation and prediction of terms from regularities in a collection of numbers, figures or pictures.
Number system	5	Digits, numbers, number systems	identifies, tells apart, and describes objects, things, and people based on properties
			recognition of similar and differing attributes, organizing into groups (find similarities, differences)
			identifies and selects elements matching multiple criteria (find similarities and regularities)
			describes the properties of numbers; describes and relates to other numbers;
			knows and recognizes Roman numerals
			Natural numbers and their properties and how numbers can be divided, and how they can be used to specify quantities and order.
			Symbols for numbers and the historical development of symbols in some different cultures through history.
			Reading, representation (including the number line and with manipulative materials), composition, decomposition, and re-composition of natural numbers up to 9999.
			Base ten number system (up to 9999): application of the relations it generates in operations.
			Deepen and enforce students' perception of the decimal numeral system (place-value).
			understands how number systems work; breaks up numbers into sums based on powers of 10;, understands place and face value
			The positional number system and how it is used to describe natural numbers.
			Model and analyze three-digit numbers and thus expand and reinforce the knowledge of place value. Introduction of the number systems and numbers used by ancient civilizations.
			Different number systems have been used in various cultures throughout history.

Math. logic and set theory	1	Categorization, Classification	Regularities: Creates statements about a given set; uses the terms "all", "not all", " exists...", "none of ...", and their synonyms appropriately
			Students improve their skills in finding similarities, differences, and regularities.
			Students improve their skills in finding similarities, differences, and regularities.
			Students improve their skills in finding similarities, differences, and regularities.
			identifies, tells apart, and describes objects, things, and people based on properties
			recognition of similar and differing attributes, organizing into groups, creating a Venn diagram
			Creates statements about a given set; uses the terms "all", "not all"," exists...", "none of ...", and their synonyms appropriately
			Students deepen their skills in comparing, classifying, and ordering, searching for answer options systematically, and observing cause and effect relationships in maths.
			Organizes into sets, takes two criteria into consideration at the same time, and names the organizing criteria
			identifies and selects elements matching multiple criteria
	18	Sets	No
	19	Math and logic in everyday life	Determines "true" and "false" statements; creates statements with the terms "all", "not all", "exists...", "none of ...", and their synonyms appropriately
	29	Mathematical language, reasoning, logic, and combinatorics	No
Probability and Statistics.	13	Data collection and management	Statistical graphs of everyday life (pictograms, bar charts, histograms...): reading, interpretation. Simple strategies for the collection, classification, and organization of discrete qualitative or quantitative data in small samples using a calculator and simple computer applications. Absolute frequency: interpretation.

			Simple statistical graphs to represent data, selecting the most convenient, using traditional resources and simple computer applications. Mode: interpretation as the most frequent data. Graphical comparison of two sets of data to establish relationships and draw conclusions.
			Students' skills to systematically collect information on interesting topics are developed.
			Students store and present information using tables and diagrams.
			collects data in their environment; records data for later evaluation; organizes collected data in a table, and illustrates it on a diagram;
			Read and interpret simple tables with at most three data groups, and organize the data obtained from the table. Examine and create the column chart. Use different representations to present the data, solve and set up problems related to daily life by using the information shown in tree diagrams, column graphs, tables, and other graphics.
	14	Randomness	Formulation of conjectures from data collected and analyzed, making sense of them in the context of the study.
			Probability as a subjective measure of uncertainty. Recognition of uncertainty in everyday situations and by performing experiments. Identification of certain events, possible events, and impossible events. Comparing the probability of two events intuitively.
			Students familiarize themselves with probability based on everyday situations by concluding whether an event is impossible, possible, or certain.
			randomness in games; understanding "certain", "Impossible", "possible but not certain"
			approximating the probability of events; counting events, comparing the approximation with the result (for example, in a game)
			Random events in specific situations.
	21	Ratios, percentages	Students familiarize themselves with the concept of percent. They ground the understanding of percentage and value and practice calculating them in simple cases.
			Use the connections between fractions, decimals, and percentages.
			Students familiarize themselves with the concept of percent. They ground the understanding of percentage and value and practice calculating them in simple cases.
			Graphs for expressing proportional relationships.

			Numbers in percentage form and their relation to numbers in fraction and decimal form.
			Proportional relationships, including doubling and halving.
			Proportionality and how proportional relationships are expressed in fraction, decimal, and percent form.
	27	Descriptive statistics	Statistics: maximum and minimum, average, and mode.
			Choosing the correct values for the frequency axis of a chart. Plan and carry out a statistical test. Read line graphs and pie charts.
			Understand the test and outcome. Describe all possible outcomes. Compare the likelihood of different outcomes. Formulate and evaluate statements about the probability of outcomes.
			Measures of central tendency – average, mode, and median, and how they are used in statistical investigations.
	28	Probability theory	Simple combinatorial analysis in concrete situations.
			Random events, chance, and risk are based on observations, simulations, and statistical data. Comparison of probability in different random trials.

Table 3. Detailed description of mathematical topics for age groups 11-12

	Age group 11-12 (grade 5-6)		
Math Topic		**Topic categories based on curriculum analysis**	**Short description of outcomes in LP based on the outcomes of the analysis**
Geometry	9	Shapes and objects, and their properties	Find similarities, differences, and regularities.
	11	Orientation in space and on a plane	Basic two- and three-dimensional geometrical objects and their properties and relationships. Construction of geometrical objects.
	25	Planar and spatial shapes' constructions, transformations, properties, and classification	Build, draw, examine, and classify shapes. Classify shapes into cylinders, cones, and other shapes.

			Classify 2D shapes into polygons and others, and study their properties.
			Study symmetry about a line.
			Learn about the concepts of point, segment, line, and angle.
			Observe rotational and translational symmetry in their surroundings
			Use the scale when using the map.
			Geometric figures in everyday objects, Geometric vocabulary, Properties of geometric figures
			Measure and calculate the perimeters and areas of different shapes and the volumes of rectangular cuboids.
			Learn more about triangles, quadrilaterals, and circles.
			groups triangles based on their angles and sides;
			Methods for determining and estimating the circumference and areas of different two-dimensional geometrical figures.
			Explain, show, and draw basic geometric concepts. Name polygons and recognize their basic elements of rectangle, parallelogram, rhombus, and trapezoid.
			Calculating the perimeters of polygons. Calculate the area of the rectangle in square centimeters and square meters, recognize the rectangular prism, determine its basic properties, draw the surface expansion, and calculate the surface area.
			Construction techniques of geometric figures by composition and decomposition, using manipulative materials, drawing instruments, and computer applications.
			Location and movements on plans and maps from reference points
			Transformations by means of rotations, translations, and symmetries. Similarity in everyday life situations
			Strategies for calculating areas and perimeters. Geometric models in solving problems, Elaboration of conjectures, and using drawing instruments.

Analysis	10	Constructions and Translations	Statistical data sets and graphs of everyday life. Strategies for conducting a simple statistical study. Absolute and relative frequency tables.
			Simple statistical graphs (bar chart, pie chart, histogram, etc.). Measures of centralization. Measures of dispersion (range).
			Calculator and other digital resources, such as a spreadsheet, to organize statistical information and perform different data visualizations. Relation and comparison of two sets of data from their graphical representation.
	23	Functional relationships	Description of positions and movements in the first quadrant of the Cartesian coordinate system.
			Learn about the first quadrant of the coordinate system and extend then to all quadrants.
			Finds their way around a coordinate system.
			Coordinate system and grading of coordinate axes.
Measuring	6	Measurements and measurement tools	Practice measuring and pay attention to accuracy, result evaluation, and checking the measurement.
			Solving problems related to responsible consumption.
			Guide the students to use the scale when using the map.
	26	Measurements and Units	Practice measuring and pay attention to accuracy, result evaluation, and checking the measurement.
			Instruments (analog or digital) and appropriate units to measure lengths, objects, angles, and times.
			Measure and calculate the perimeters and areas.
Algebra	2	Problem solving	Solve movement problems using diagrams and various models.

	3	Comparison, sorting	Students deepen their skills in comparing, classifying, and ordering, searching for answer options systematically, and observing cause and effect relationships in maths.
			Complete a figure to make it symmetrical, and re-create a symmetrical figure. Represent transformations. Resizing of a shape.
	4	Counting, approximations	No
	7	Equations, operations	Equality is the relationship between two elements and obtaining simple unknowns (represented by a symbol) in either element.
			Define and use simple, compound, and integer fractions, and addition and subtraction of fractions. Add and subtract fractions with equal denominators and solve appropriate problems.
			Perform four operations on natural numbers.
			Addition and subtraction operations with these fractions, and make sense of these operations. Decimal notation with fractions and perform addition and subtraction.
			Methods, including algebraic, for solving simple equations.
	8	Mental calculations	Students deepen their skills in comparing, classifying, and ordering, searching for answer options systematically, and observing cause and effect relationships in maths.
			Students are guided to round numbers and calculate with approximations so that they learn to estimate the order of magnitude of the result.
	12	Relationships	Students deepen their skills in comparing, classifying, and ordering, searching for answer options systematically, and observing cause and effect relationships in maths.

			Understand that operations with decimal numbers are similar to those with whole numbers. Additionally, addition, subtraction, multiplication, and division with rational numbers are visualized and justified.
			Patterns in number sequences and geometrical patterns, and how they are constructed, described, and expressed.
	15	Columnar operations	Practice addition and subtraction algorithms, ensuring that the skill is learned. (columnar addition and subtraction)
			Practice the multiplication algorithm and ensure that the skill is mastered (columnar multiplication).
	16	Part-Part-Whole Relationships	Learn the concept of fractions and practice basic calculations of fractions in different situations.
	17	Natural numbers, Integers, and their operations	Ground the concept of a negative number and expand the number range with negative integers.
			Varied counting strategies, systematic counting, and adapting counting to the size of numbers.
			Strategies and techniques for interpreting and manipulating the order of magnitude of numbers.
			Guided to round numbers and calculate with approximations, learn to estimate the order of magnitude of the result.
			When solving practical problems, divide by at most 2-digit numbers in writing. Approximates the quotient.
			Compare and round natural numbers using different methods. Define a coordinate plane, and understand how pairs of numbers represent a point on it.
			The four basic arithmetic operations (addition, subtraction, multiplication, and division) and rules for their use in calculations with natural numbers.
			Perform four operations on natural numbers.

			Determines the given numbers' negative, absolute value; knows integers.
	20	Rational numbers, fractions, and their operations	Reading, representation, composition, decomposition, and re-composition of natural numbers and decimals to thousandths. Fractions and decimals to express quantities and choosing the best representation for each situation or problem.
			Base ten numbering system (natural numbers and decimals to thousandths): application of the relations it generates in operations. Natural numbers, fractions, and decimals to thousandths in everyday life contexts: comparison and ordering.
			Familiarize themselves with decimal numbers as part of the decimal system and practice basic calculations with decimal numbers.
			Can illustrate fractional parts, can determine fractions corresponding to a fractional part;
			Knows and utilizes place value notation for decimal fractions;
			Rational numbers, including negative numbers, and their properties, and how the numbers can be divided and used.
			The positional number system and how it is used to describe whole numbers and decimal numbers.
			Associate decimal notation with fractions and perform addition and subtraction.
			Make sense of integer and compound fractions, conversions, numerators, or fractions with equal denominators, and fractions whose denominator is a multiple of the denominator of the other
			Learn the concept of fractions and practice basic calculations of fractions in different situations.
			Strengthen arithmetic skills with fractions and learn how to multiply and divide fractions by fractions.
			Get acquainted with the concepts of opposite number, reciprocal, and absolute value.

			Determines the reciprocal of given numbers.
			Methods for calculations with natural numbers, simple fractions, and decimals in approximate estimates, mental arithmetic, and written calculations. Use digital tools.
			Mental calculation strategies with natural numbers, fractions, and decimals. Strategies for recognizing which simple or combined operations (+, -, *, /) are useful to solve contextualized situations. Strategies for solving arithmetic operations (with natural numbers, decimals, and fractions) with flexibility and sense: mentally, in writing, or with a calculator; usefulness in contextualized situations and properties.
			Familiarize themselves with decimal numbers as part of the decimal system and practice basic calculations with decimal numbers.
			Deepen the knowledge of calculating with decimal numbers.
			During practical problems, divide a decimal fraction with at most two-digit whole numbers in writing. Approximates the quotient.
			Understand that operations with decimal numbers are similar to those with whole numbers. Additionally, addition, subtraction, multiplication, and division with rational numbers are visualized and justified.
			How numbers in fractions and decimals can be used in everyday situations.
			Addition and subtraction operations with these fractions, and make sense of these operations. Decimal notation with fractions and perform addition and subtraction.
	22	Problem-solving with equations, proofing	Equality and inequality relations and use of the signs < and >. Determination of unknown data (represented by a letter or symbol) in simple expressions related by means of these signs and the signs = and ?.
			Study equations and find solutions by reasoning and experimenting.
			Study equations and find solutions by reasoning and experimenting.
			Interprets and double-checks the result.

			First-degree equations are solved. Real-world problems with direct proportionality are discussed. Ratio and proportionality are defined. Properties of proportions are understood and used to solve problems.
			Formulation of mathematical questions based on everyday situations.
			Strategies for solving mathematical problems in student-related situations.
			Formulate simple mathematical conjectures by investigating patterns, properties, and relationships in a guided manner. Pose new problems that are solved mathematically.
			Use connections between different mathematical elements by mobilizing one's own knowledge and experiences. Use connections between mathematics.
			The process of modelling everyday problems using mathematical representations.
			Select among different strategies to solve a problem, justifying the choice. Obtain possible solutions to a problem, selecting among several known strategies in an autonomous way. Verify the mathematical correctness of the solutions to a problem and their coherence in the given context.
			Solves word problems through deduction or equations; approximates.
			Create and re-arrange simple alphabetic expressions using natural numbers.
			Methods, including algebraic, for solving simple equations.
			Linear and quadratic relationships in everyday or mathematically relevant situations: expression using symbolic algebra.
			Formulate and check simple conjectures in a guided way, analyzing patterns, properties, and relationships. Propose variants of a given problem by modifying some of its data or some condition of the problem. Use appropriate technological tools in the investigation and verification of conjectures or problems.

			Recognize situations that can be formulated and solved using mathematical tools and strategies, establishing connections between the real world and mathematics, and using the processes inherent to research: inferring, measuring, communicating, classifying, and predicting. Identify coherent connections between mathematics and other subjects by solving contextualized problems.
			Interpret mathematical problems by organizing data, establishing relationships between them, and understanding the questions asked. Apply appropriate tools and strategies. Obtain mathematical solutions to a problem, activating knowledge and using the necessary technological tools.
			Recognize the relationships between mathematical knowledge and experiences, forming a coherent whole. Make connections between different mathematical processes by applying prior knowledge and experiences.
			Modelling of everyday situations using mathematical representations and algebraic language.
			Represent mathematical concepts, procedures, information, and results in different ways and with different tools, including digital tools, visualizing ideas, structuring mathematical processes, and valuing their usefulness for sharing information. Elaborate mathematical representations that help in the search for strategies to solve a problematized situation.
			Equivalence of algebraic expressions in solving problems based on linear and quadratic relationships.
			Form and solve first-order equations and incomplete quadratic equations.
			Learn about the concept of polynomials and practice addition, subtraction, and multiplication of polynomials.

			Students familiarize themselves with the concept of a variable and calculating the value of an expression. They practice forming expressions and simplifying them.
			With simple lettered expressions, calculates addition, subtraction, and substitution value;
			Solves one variable, first-degree equations using different methods;
			Defining the concepts of unary, binary, ternary, and polynomial. Multiplying alphabetic expressions
			Solve problems that require you to select solutions to inequalities that satisfy certain conditions.
			Dealing with various real-world situations that can be modelled by systems of equations.
			Formulation of mathematical questions based on different situations and subject areas.
			Linear equation and rate of change. Using the linear equation to describe relationships.
			Strategies for solving mathematical problems in different situations and within different subject areas, as well as the evaluation of chosen strategies and methods.
			Variables and their use in simple algebraic expressions and equations.
	24	Series (Sequences)	Studying the regularity of number sequences and continuing number sequences according to a rule.
			Deepen the students' ability to study and form number sequences.
			Studying the regularity of number sequences and continuing number sequences according to a rule.
			Continues periodic series based on a given rule; recognizes and describes the generating rule of a series given by a few of its members.

			Solve problems in a variety of contexts where different ways of describing number sequences are considered, applied, and combined. Problem situations are addressed by identifying gaps in mathematical information and learning how to find and retrieve it.
			Number sequences are examined where the sequence is generated by an operation or a sequence of operations.
	30	Number theory, LCM, GCD, power, root	Relationships between arithmetic operations: application in everyday contexts. Relationship of divisibility: multiples and divisors.
			Students familiarize themselves with the divisibility of numbers and divide numbers into prime factors.
			knows and uses the divisibility rules for 2, 3, 4, 5, 6, 9, 10, 100; groups numbers based on the number of divisors or remainders
			Power is a product of equal factors. Squares and cubes.
	31	Pattern usage/recognition	Creation of recurring patterns from regularities or other patterns using numbers, figures, or images.
			Strategies for identification, oral description, discovery of hidden elements, and extension of sequences from regularities in a collection of numbers, figures, or images.
			Identification, verbal description, representation, and reasoned prediction of terms from regularities in a collection of numbers, figures, or pictures.
			Strategies for identifying, representing (verbally or by means of tables, graphs, and invented notations), and reasoning about terms from regularities in a collection of numbers, figures, or pictures.
			Strategies for reasoned prediction of terms from regularities in a collection of numbers, figures, or pictures.
			Recognize patterns facilitating its computational interpretation.
			Strategies for representing terms from regularities in a collection of numbers, figures, or pictures.

Number system	5	Digits, numbers, number systems	Understands and uses the place value notation of large numbers;
			Different number systems have been used in various cultures throughout history.
			Read and write natural numbers
Math, logic, and set theory	1	Categorization, Classification	No
	18	Sets	Recognize sets in concrete cases
			Illustrate sets in concrete cases
	19	Math and logic in everyday life	Understand problems of everyday life. Elaborate mathematical representations that help in the search and choice of strategies and tools, including technological ones, for the resolution of a problematized situation.
			Interpret simple mathematical language present in everyday life in different formats, acquiring appropriate vocabulary, and showing understanding of the message. Communicate mathematical conjectures and processes in different formats, using appropriate mathematical language.
			Activities that require logical thinking are practiced, such as looking for rules and dependencies and presenting them precisely. Consider and determine the number of options (in math problems).
			Practice interpreting and producing mathematical text.
			Strengthen the students' skills in reasoning and justification.
			determines the logical value (true or false) of statements.
			Understand that there are different ways of justifying a statement. Not all justifications are mathematical proof.

	29	Mathematical language, reasoning, logic, and combinatorics	No
Probability and Statistics.	13	Data collection and management	Students' skills to systematically collect information on interesting topics are developed.
			Create and interpret simple tables and bar graphs.
			Create and interpret simple tables and bar graphs.
			Students store and present information using tables and diagrams.
	14	Randomness	Students familiarize themselves with probability based on everyday situations by concluding whether an event is impossible, possible, or certain.
			Randomness in games: understanding "certain", "Impossible", "possible but not certain"
			Approximating the probability of events, counting events, and comparing the approximation with the result (for example, in a game)
			Handling charts with large amounts of data. Probabilistic games were discussed and developed, where all players have the same chance of winning. Drawing charts and data tables, finding numerical characteristics, digital technologies are used.
	21	Ratios, percentages	Strategies for comparing and ordering measurements of the same magnitude, applying the equivalences between units. Relationship between the decimal metric system and the decimal numbering system. Estimation of angle and surface measurements by comparison. Evaluation of results of measurements and estimations or calculations of measurements, reasoning.
			Conventional units of the Decimal Metric System (length, mass, capacity, volume, and surface area), time, and degree (angles) in everyday life contexts: selection and use of appropriate units.
			Instruments (analog or digital) and appropriate units to measure lengths, objects, angles, and times.
			Solving problems related to responsible consumption and money.

			Numerical information in simple financial contexts. Methods for making responsible consumption decisions.
			Proportional and non-proportional situations in everyday life problems. Solving problems of proportionality, percentages, and scales through the equality between ratios, reduction to unity, or the use of proportionality coefficients.
			Ratios and proportions: understanding and representation of quantitative relationships. Percentages: understanding and solving problems. Proportionality situations in different contexts: analysis and development of methods for solving problems (percentage increases and decreases, price reductions and increases, taxes, scales, currency exchange, speed and time, etc.).
			Percentages greater than 100 and less than 1.
			Relationship between simple fractions, decimals, and percentages.
			Comparison and ordering of fractions, decimals, and percentages: exact or approximate location on the number line.
			In addition, learn how to calculate the changed value, basic value, and change and comparison percentage.
			Students familiarize themselves with the concept of percent. They ground the understanding of percentage and value and practice calculating them in simple cases.
			Use the connections between fractions, decimals, and percentages.
			Ensure the understanding of the concept of percent.
			Practice calculating the percentage and calculating the amount indicated by the percentage from the whole.
			Students familiarize themselves with the concept of percent. They ground the understanding of percentage and value and practice calculating them in simple cases.
			Understands the concept of percentage; solves economic, finance, and everyday life problems related to percentages;
			Exchanges units of measurement regarding time, mass, length, area, and volume based on decimal thinking, with practical considerations.
			Percentage is used in real-world problems about buying, selling, and discounts. Financial decisions are based on calculations.

			Examines input/output (I and/or O) tables expressing the inverse proportionality relation, learning how to construct such tables and relate them to the problem condition. Solve problems in a variety of contexts where quantities are related by a linear relationship.
			Graphs for expressing proportional relationships.
			Numbers in percentage form and their relation to numbers in fraction and decimal form.
			Proportional relationships, including doubling and halving.
			Proportionality and how proportional relationships are expressed in fraction, decimal, and percent form.
			Proportionality and how it is used to express scale, uniformity, and change.
			Percentage and change factor to express change, as well as calculations with percentages, in everyday situations and in different subject areas.
			The concept of percentage, the concept of percent, is associated with fraction and decimal notations.
	27	Descriptive statistics	Identification of a data set as a sample of a larger set and reflection on the population to which it is possible to apply the conclusions of simple statistical investigations.
			Formulation of appropriate questions that allow for the knowledge of the characteristics of interest of a population. Relevant data to answer questions posed in statistical investigations. Strategies for drawing conclusions from a sample in order to make judgments and appropriate decisions.
			Statistical data sets and graphs of everyday life: description, interpretation, and critical analysis. Strategies for conducting a simple statistical study: formulation of questions, and collection, recording, and organization of qualitative and quantitative data from different experiments (surveys, measurements, observations ...). Absolute and relative frequency tables: interpretation.
			Simple statistical graphs. Measures of centralization (mean and mode). Measures of dispersion (range).
			Calculator and other digital resources, such as a spreadsheet, to organize statistical information and perform different data visualizations. Relation and comparison of two sets of data from their graphical representation: formulation of conjectures, analysis of dispersion, and drawing conclusions.

			Strategies for collecting and organizing data from daily life situations involving a single variable. Difference between variable and single values. Analysis and interpretation of statistical tables and graphs of qualitative, discrete, quantitative, and continuous variables in real contexts. Statistical graphs: representation using different technologies (calculator, spreadsheet, applications...) and choice of the most appropriate one.
			Location measures: interpretation and calculation with technological support in real situations. Variability: interpretation and calculation, with technological support, of dispersion measures in real situations. Comparison of two data sets according to location and dispersion measures.
			Ensure understanding of the average value and type value.
			Deepen students' skills in collecting, structuring, and analyzing information.
			Practice determining frequency, relative frequency, and median.
			Use proportion to solve problems.
			Statistics: maximum and minimum, average, and mode.
			interprets data in tables, selects the appropriate visualization method, and creates the visualization;
			translates different diagrams into each other;
			collects data according to a given requirement from a table, reads from traditional or digital diagrams, and, after organization of the data, expresses conclusions;
			calculates the average of a data series, determines the most common value (mode), the middle datapoint (median), and compares these.
			Choosing the correct values for the frequency axis of a chart. Plan and carry out a statistical test. Read line graphs and pie charts.
			Understand the test and outcome. Describe all possible outcomes. Compare the likelihood of different outcomes. Formulate and evaluate statements about the probability of outcomes.
			Measures of central tendency – average, mode, and median, and how they are used in statistical investigations.
	28	Probability theory	Uncertainty in everyday life situations. Calculation of probabilities in experiments, comparisons or investigations in which Laplace's rule is applicable.

			Deterministic and random phenomena: identification. Simple experiments. Assignment of probabilities by experimentation, the concept of relative frequency, and Laplace's rule.
			Various diagrams are interpreted and created.
			Calculate the probabilities.
			Get acquainted with the concept of standard deviation.
			Plays probabilistic games, performs probabilistic experiments, and through these collects, organizes, and displays the data according to a plan, also digitally.
			Understands the concepts of frequency and relative frequency. Uses this knowledge to explain "impossible", "certain", "less/more likely" statements.
			Explaining the nature of different types of data and how variability in datasets can be interpreted in practice. Explaining how cumulative frequency and cumulative relative frequency table data are represented by a cumulative frequency or cumulative relative frequency chart, and how to read and interpret the data represented by such charts.
			Two-outcome feasibility/probability trees and tables are constructed
			Simple combinatorial analysis in concrete situations.
			Measures of central tendency, measures of dispersion, and how they are used for assessing results of statistical studies.
			Random events, chance, and risk are based on observations, simulations, and statistical data. Comparison of probability in different random trials.

Table 4. Detailed description of mathematical topics for age groups 13-14

	Age group 13-14 (grade 7-8)		
Math Topic		**Topic categories based on curriculum analysis**	**Short description of outcomes in LP based on the outcomes of the analysis**
Geometry	9	Shapes and objects, and their properties	No
	11	Orientation in space and on a plane	No
	25	Planar and spatial shapes' constructions, transformations, properties, and classification	Classification of planar and spatial shapes
			Create Venn-diagram
			Calculates perimeter and area of polygons and other planar shapes
			Apply the Pythagorean theorem for problem-solving
			Geometric modelling: numeric and algebraic relationships in problem solving
			Congruence and similarity => proportion, multiplication
			Proving congruence with the sequence of transformation... (like the transposition method for solving equations)
			symmetry (congruence - the neutral item of transformation!)
Analysis	10	Constructions and Translations	No
	23	Functional relationships	Quantitative relationships in everyday life situations and the kinds of functions that model them
			Selection of the appropriate representation for the same quantity

			Linear and quadratic relationships: identification and comparison
			Describe dependencies both graphically and algebraically (direct and indirect proportionality)
			Interpret graphs (increase and decrease of a function, slope and constant term, zeros of a function)
			Maps the elements of two concrete sets to each other
			Illustrates the data in a data table graphically
Measuring	6	Measurements and measurement tools	No
	26	Measurements and Units	Estimation of measurement
			Measurable attributes of physical and mathematical objects
			Skills in units of measurement and their conversions
			Surface and volume calculation
Algebra	2	Problem solving	No
	3	Comparison, sorting	No
	4	Counting, approximations	No
	7	Equations, operations	No
	8	Mental calculations	No
	12	Relationships	No
	15	Columnar operations	No
	16	Part-Part-Whole Relationships	No
	17	Natural numbers, Integers, and their operations	Varied systematic counting strategies in everyday life
			Adaptation of counting to the size of numbers
			Exact value, approximate value, and rounding

			Operations with negative numbers
	20	Rational numbers, fractions, and their operations	arithmetic of fractions
			opposite number, reciprocal value, absolute value
			operations with decimal numbers
	22	Problem-solving with equations, proofing	Problem-solving with equations (approximation, decomposition or transposition method)
			Express linear and quadratic relationships with symbolic algebra
			Modelling of everyday situations using mathematical representations and algebraic language
			Equivalence of algebraic expressions (linear and quadratic)
			Form and solve first-order equations and incomplete quadratic equations
			Operations of polynomials
			Forming and simplifying expressions
			Select solutions to inequalities that satisfy certain conditions
			Strategies for solving mathematical problems and evaluation of chosen strategies and methods
			Variables and their use in algebraic expressions, formulae, equations, and functions
			Decompose a problem into simpler parts, facilitating its computational interpretation
	24	Series (Sequences)	Numerical patterns and regularities
			Form number sequences
			Constructing, describing, and expressing patterns in number sequences and geometrical patterns
	30	Number theory, LCM, GCD, power, root	Divisibility of numbers
			Divide numbers into prime factors

			Calculates the lowest common denominator and the greatest common divisor
			Power calculations with an integer exponent
			Simplifying power expressions
			The square root of square numbers
	31	Pattern usage/recognition	Recognition of patterns facilitates its computational interpretation
Number system	5	Digits, numbers, number systems	No
Math, logic, and set theory	1	Categorization, Classification	No
	18	Sets	Sorts elements into sets based on multiple criteria
			Subset recognition and illustration
			Numbers, sets of numbers illustration
			Set operations (complement, intersection, union)
			Set of rational numbers, infinite non-periodic decimal fractions
			Set of real numbers, properties, and usage
	19	Math and logic in everyday life	Interpreting and producing mathematical text
			Reasoning and justification
			Looking for rules and dependencies and presenting them precisely
			Consider and determine the number of options (in math problems)

	29	Mathematical language, reasoning, logic, and combinatorics	Using appropriate mathematical language
			Describing, explaining, and justifying reasoning, procedures, and conclusions
			Strategies for deducing reasonable conclusions from a mathematical model
			Deduces truth values for propositions
			Expresses true and false statements
			Methods for overviewing all of the cases (in calculations)
			Illustrates tasks in graphs (Graph theory, Seven Bridges of Konigsberg)
			Usage of combinatorial principles
Probability and Statistics.	13	Data collection and management	No
	14	Randomness	No
	21	Ratios, percentages	Understanding and representation of quantitative relationships
			Comparison of decimals and percentages
			Understanding of the concept of percent
			Calculate the amount indicated by the percentage of the whole
			Solve economic, finance, and everyday life problems related to percentages
			Exchanges units of measurement regarding time, mass, length, area, and volume based on decimal thinking
			Problem-solving by a linear and inverse relationship
			Identify the multiplicities given the ratios
	27	Descriptive statistics	Relevant data to answer questions posed in statistical investigations
			Strategies for drawing conclusions from a sample in order to make judgments and appropriate decisions

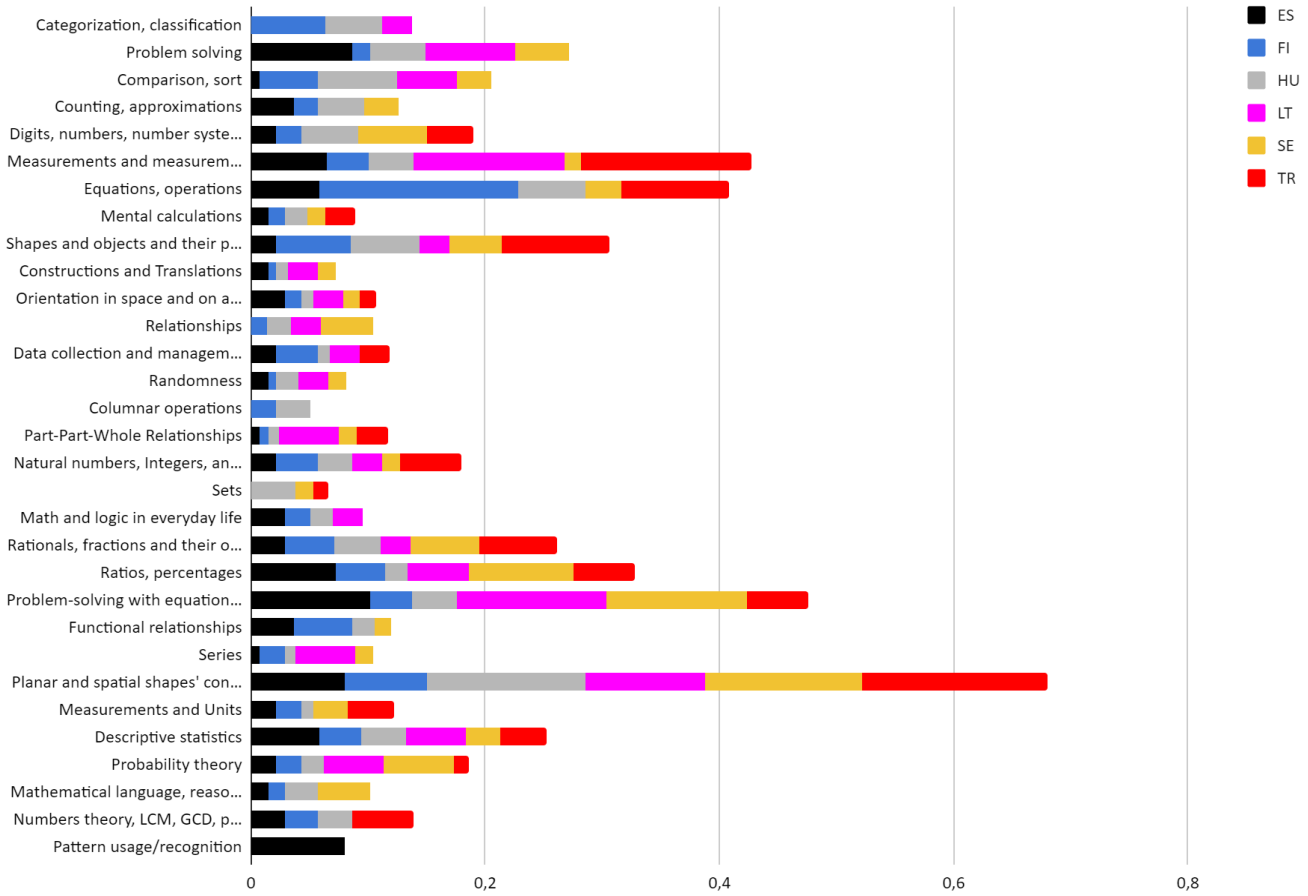
			Strategies for collecting and organizing data for a single variable
			Analysis and interpretation of statistical tables and graphs of qualitative, discrete quantitative, and continuous quantitative variables
			Understanding of the average value
			Determining frequency, relative frequency, and median
			Problem-solving with proportion
			Interprets data in tables, selects the appropriate visualization method, and creates the visualization
			Calculates the average of a data series, determines the most common value (mode), the middle datapoint (median), and compares these
	28	Probability theory	Identify deterministic and random phenomena
			Simple experiments: planning, performance, and analysis of the associated uncertainty
			Assignment of probabilities to experiments
			Calculate the probabilities
			Calculate the standard deviation
			Explain impossible, certain, less/more likely statements

4. Summary

Our detailed analysis of mathematics education literature allowed us to define cognitive development areas related to Algebraic Thinking (AT) as described in the research literature. We applied this definition, along with a higher-order definition of Computational Thinking (CT), to code statements in the mathematics curricula of six European nations. The analysis revealed substantial similarities but also notable differences, with a statistical analysis showing close to 50% congruence among the core curricula.

A general learning path emerges, starting from simple objects and their classification and categorization, moving through the investigation of properties and relations, recognizing patterns, and generalizing. As learners master basic numerals, they begin to use symbols for numbers and operations. Their mathematical vocabulary expands, leading to problem-solving with advanced integrative concepts and definitions. This general learning path covers major components of AT and will be used in the next project stage to develop tasks supporting individual learning trajectories within each path. Individualization will be achieved by applying learning analytics to define personalized trajectories based on prior task performance and achievements.

Statistics


Categorization, classification
Problem solving
Comparison, sort
Counting, approximations
Digits, numbers, number syste...
Measurements and measurem...
Equations, operations
Mental calculations
Shapes and objects and their p...
Constructions and Translations
Orientation in space and on a...
Relationships
Data collection and managem...
Randomness
Columnar operations
Part-Part-Whole Relationships
Natural numbers, Integers, an...
Sets
Math and logic in everyday life
Rationals, fractions and their o...
Ratios, percentages
Problem-solving with equation...
Functional relationships
Series
Planar and spatial shapes' con...
Measurements and Units
Descriptive statistics
Probability theory
Mathematical language, reaso...
Numbers theory, LCM, GCD, p...
Pattern usage/recognition
0
0,2
0,4
0,6
0,8
ES
FI
HU
LT
SE
TR

Composition:	1, 3, 6, 9, 18, 22, 23, 26, 27	1, 3, 6, 7, 9, 18, 22, 26, 27	1, 3, 6, 7, 9, 10, 11, 12, 16, 17, 18, 19, 20, 21, 22, 23, 25, 26, 27	2, 6, 9, 10, 11, 12, 13, 14, 17, 19, 21, 22, 23, 24, 25, 28, 29, 30, 31	2, 9, 10, 11, 12, 13, 14, 23, 24, 25, 26, 27, 28, 29, 30, 31	9, 14, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 27, 28, 29, 30, 31	2	4, 5, 6, 7, 8, 10, 13, 14, 15, 16, 17, 20, 21, 22, 23, 24, 25, 26, 27, 28, 30, 31	2, 3, 5, 6, 7, 23		
Totals:	127	134	324	266	238	228	3	352	37		AT categories used in this given group
	Equality	Inequality	Relational thinking	Problem solving	Patterns	Generalization	Variables / Unknowns	Numbers	Symbols	Total	
Categorization, classification	15	15	15	0	0	0	0	0	0	45	Equality, Inequality, Relational thinking
Problem solving	0	0	0	22	1	0	3	0	1	27	Problem solving, Patterns, Variables / Unknowns, Symbols
Comparison, sort	19	19	19	0	0	0	0	0	1	58	Equality, Inequality, Relational thinking, Symbols
Counting, approximations	0	0	0	0	0	0	0	14	0	14	Numbers
Digits, numbers, number systems	0	0	0	0	0	0	0	9	9	18	Numbers, Symbols
Measurements and measurement tools	5	12	12	1	0	0	0	18	2	50	Equality, Inequality, Relational thinking, Problem solving, Numbers, Symbols
Equations, operations	0	5	7	0	0	0	0	42	9	63	Inequality, Relational thinking, Numbers, Symbols
Mental calculations	0	0	0	0	0	0	0	9	0	9	Numbers
Shapes and objects and their properties	2	2	20	8	26	3	0	0	0	61	Equality, Inequality, Relational thinking, Problem solving, Patterns, Generalization
Constructions and Translations	0	0	2	6	6	0	0	4	0	18	Relational thinking, Problem solving, Patterns, Numbers

Orientation in space and on a plane	0	0	10	10	10	0	0	0	0	30	Relational thinking, Problem solving, Patterns
Relationships	0	0	8	8	6	0	0	0	0	22	Relational thinking, Problem solving, Patterns
Data collection and management	0	0	0	12	12	0	0	12	0	36	Problem solving, Patterns, Numbers
Randomness	0	0	0	4	7	3	0	4	0	18	Problem solving, Patterns, Generalization, Numbers
Columnar operations	0	0	0	0	0	0	0	6	0	6	Numbers
Part-Part-Whole Relationships	0	0	1	0	0	3	0	8	0	12	Relational thinking, Generalization, Numbers
Natural numbers, Integers, and their operations	0	0	10	4	0	8	0	17	0	39	Relational thinking, Problem solving, Generalization, Numbers
Sets	6	6	6	0	0	6	0	0	0	24	Equality, Inequality, Relational thinking, Generalization
Math and logic in everyday life	0	0	10	10	0	10	0	0	0	30	Relational thinking, Problem solving, Generalization
Rationals, fractions, and their operations	0	0	24	0	0	24	0	24	0	72	Relational thinking, Generalization, Numbers
Ratios, percentages	0	0	30	30	0	30	0	30	0	120	Relational thinking, Problem solving, Generalization, Numbers
Problem-solving with equations, proofing	40	39	39	22	0	13	0	40	0	193	Equality, Inequality, Relational thinking, Problem solving, Generalization, Numbers

Functional relationships	4	0	15	15	15	8	0	5	15	77	Equality, Relational thinking, Problem solving, Patterns, Generalization, Numbers, Symbols
Series	0	0	0	3	8	7	0	8	0	26	Problem solving, Patterns, Generalization, Numbers
Planar and spatial shapes' constructions, transformations, properties, and classification	0	0	60	60	60	38	0	37	0	255	Relational thinking, Problem solving, Patterns, Generalization, Numbers
Measurements and Units	12	12	12	0	12	0	0	12	0	60	Equality, Inequality, Relational thinking, Patterns, Numbers
Descriptive statistics	24	24	24	0	24	24	0	21	0	141	Equality, Inequality, Relational thinking, Patterns, Generalization, Numbers
Probability theory	0	0	0	15	15	15	0	6	0	51	Problem solving, Patterns, Generalization, Numbers
Mathematical language, reasoning, logic and combinatoric	0	0	0	10	10	10	0	0	0	30	Problem solving, Patterns, Generalization
Numbers theory, LCM, GCD, power, root	0	0	0	15	15	15	0	15	0	60	Problem solving, Patterns, Generalization, Numbers
Pattern usage/recognition	0	0	0	11	11	11	0	11	0	44	Problem solving, Patterns, Generalization, Numbers

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