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Learning Paths

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Aims and goals

Algebraic thinking (AT) and computational thinking (CT) share similarities in their approaches to problem-solving and in dealing with abstract concepts and symbols. Algebraic thinking focuses on using mathematical notation and symbols to represent and solve problems, while computational thinking involves breaking down complex issues into smaller, manageable steps that can be solved using algorithms and programming logic. Both require abstract reasoning, logical thinking, and effective problem-solving strategies.

For example, in algebraic thinking, students use symbolic notation to solve equations. In contrast, computational thinking encourages students to apply programming concepts to develop algorithms. Both areas emphasize the importance of recognizing patterns and relationships among different components of a problem.

Integrating algebraic and computational thinking allows students to gain a more comprehensive understanding of mathematics and its applications across various fields. This integration enables learners to apply algebraic concepts to programming and data analysis, and vice versa, enhancing their critical thinking and problem-solving skills. Additionally, it helps students develop a better intuitive understanding of abstract concepts and strengthens their ability to reason logically and systematically.

Kajsa Bråting and Cecilia Kilhamn (2021) explored the relationship between algebraic thinking and computational thinking within mathematics education. They argue that there is significant overlap between these two areas and that integrating them into the math curriculum can help students gain a more comprehensive understanding of mathematics. Their article outlines the concepts of algebraic and computational thinking, including their definitions and characteristics.

They provide examples of how to integrate these areas in math instruction, such as using algebraic expressions to represent computer programs or employing coding to explore algebraic concepts. The authors assert that combining algebraic and computational thinking in math education enhances problem-solving skills and deepens students' understanding of abstraction. However, they also highlight the challenges this approach poses for teachers, who must possess a strong understanding of both areas and have access to the necessary resources to We support this integration.

Based on our research, we conclude that combining algebraic and computational thinking in math education has significant potential to enhance students' understanding of mathematics while also developing their critical thinking and problem-solving skills. However, further research is needed to explore the effectiveness of this integration and to create more concrete strategies and resources for math teachers.

To advance this research, we have developed a categorization of concepts. Initially used for analyzing and comparing curricula, this categorization will now serve to structure and design

learning pathways. To evaluate our prototype, we have created a comprehensive bank of task items informed by the conceptual cluster structures we developed (Figure 1).

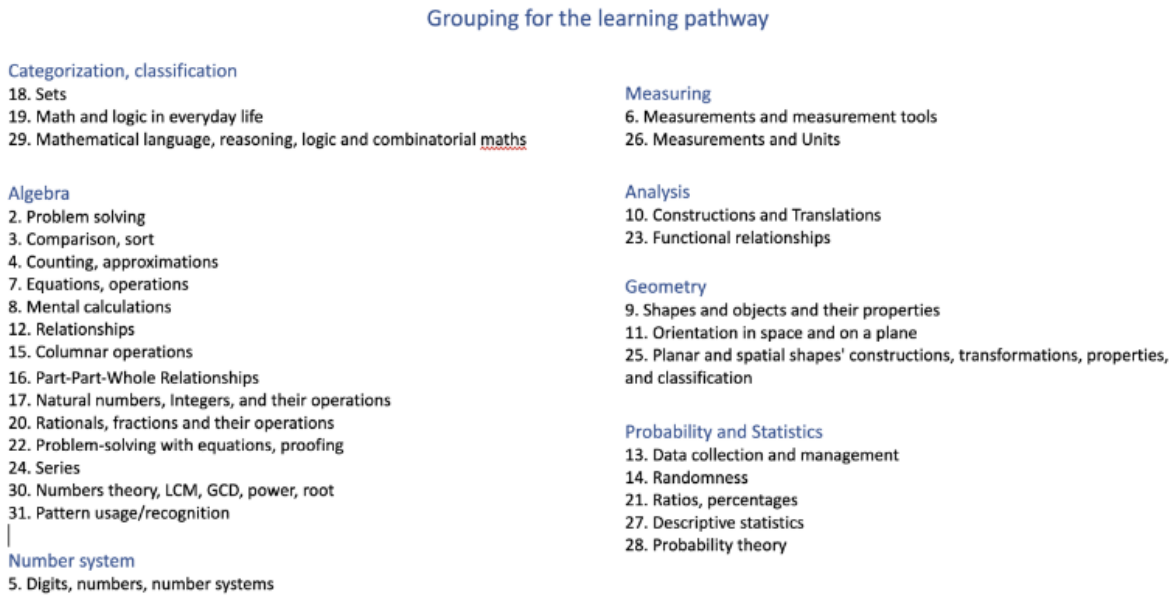

Fig.1: Grouping math categories based on Learning Outcomes

Our evaluation of these pathway prototypes draws on the work of Weintrop et al. (2016), who conclude that computational thinking should be integrated into mathematics education through activities involving algorithmic thinking, data analysis, and modeling. For example, students can use spreadsheets or programming environments to analyze data sets or create models of mathematical concepts. Students can also be given problems that require them to develop and test algorithms or use abstraction to represent mathematical concepts.

To assess computational thinking in mathematics, Weintrop et al. suggest using tasks that require students to apply computational thinking strategies to solve mathematical problems. They also recommend using rubrics that assess different aspects of computational thinking, such as problem decomposition, algorithm design, and debugging. These areas are the focus of ongoing learning analytics research in the ViLLE team.

Learning outcomes - learning path

We prepared a cross-table (Table 1) for the learning paths, based on the main categories and joined to the learning outcomes.

Table 1. The learning paths

ID	Grade	Math Area	Topic ID and Category		Serial No	
ID	**Grade**	**Math Area**	**No**	**Topic categories**	**SN**	**Short description of outcomes in the Learning paths based on outcomes of content analysis (MathFull3 sheet)**
1	3-4	Geometry	9	Shapes and objects, and their properties	34_09_01	Freely or from an example builds and constructs shapes and patterns from given objects, two-dimensional shapes; recognizes and is able to continue a line pattern or planar pattern.
2	3-4	Geometry	9	Shapes and objects, and their properties	34_09_02	Constructs bodies from edges and faces; prepares edge frames and nets of objects; finds all fitting multiple criteria, symmetry.
3	3-4	Geometry	9	Shapes and objects, and their properties	34_09_03	Basic two- and three-dimensional geometrical objects and their properties and relationships. Construction of geometrical objects, both with and without digital tools.
4	3-4	Geometry	9	Shapes and objects, and their properties	34_09_04	Realize that shapes such as squares and rectangles have more than one line of symmetry. Complete the given figure according to the horizontal or vertical line of symmetry, Covering, drawing the covering pattern on dotted or squared paper. Express more abstract concepts such as point, line, ray, line segment, and give examples from their surroundings by recognizing the angle.
5	3-4	Geometry	9	Shapes and objects, and their properties	34_09_05	Find a certain geometric pattern through experience. Determine the relationship in a pattern whose elements are objects, geometric shapes, or other objects, and finding the missing element is included.

						Create geometric patterns with at most three elements.
6	3-4	Geometry	9	Shapes and objects, and their properties	34_09_06	Simple two-dimensional geometric figures in everyday objects: identification and classification according to their elements. Basic geometric vocabulary: verbal description of the elements and properties of simple geometric figures. Properties of two-dimensional geometric figures: exploration using manipulative materials and digital tools.
7	3-4	Geometry	9	Shapes and objects, and their properties	34_09_07	Geometric figures of two or three dimensions in everyday objects: identification and classification according to their elements and the relationships between them. Vocabulary: verbal description of the elements and properties of simple geometric figures.
8	3-4	Geometry	9	Shapes and objects, and their properties	34_09_08	Build, draw, examine, and classify shapes. Classify shapes into cylinders, cones, and other shapes. The students are guided to identify and name the qualities of shapes, and they classify shapes using these.
9	3-4	Geometry	9	Shapes and objects, and their properties	34_09_09	Students develop their ability to visualize a three-dimensional environment and observe plane geometry in it.
10	3-4	Geometry	9	Shapes and objects, and their properties	34_09_10	Study symmetry about a line.
11	3-4	Geometry	9	Shapes and objects, and their properties	34_09_11	Differentiates between objects and two-dimensional figures; identifies, tells apart, and describes objects, things, people based on properties; names the properties, curves, shapes, straight lines; recognizes reflexively symmetrical and not reflexively symmetrical shapes; rectangle, square, rectangular cuboid, cube properties and differences; Learning to describe the position of these shapes relative to each other.

12	3-4	Geometry	9	Shapes and objects, and their properties	34_09_12	Basic geometrical two-dimensional objects, spheres, cones, cylinders, and cuboids. Properties of these objects and their relationships. Construction of geometric objects.
13	3-4	Geometry	9	Shapes and objects, and their properties	34_09_13	The symmetry in everyday life and how symmetry can be constructed.
14	3-4	Geometry	9	Shapes and objects, and their properties	34_09_14	Classify shapes according to the number of corners and sides, to name triangles, squares, rectangles, and circles, to recognize them, and to create models. Circle. Other shapes are expected to be classified according to the number of sides and corners. Create, draw shape models using a single known shape and different shapes. Recognize and model geometric objects.
15	3-4	Geometry	9	Shapes and objects, and their properties	34_09_15	The faces, corners, and edges. Determine the similarities and differences between cube, square, and rectangular prisms.
16	3-4	Geometry	9	Shapes and objects, and their properties	34_09_16	Using a ruler to draw triangles, squares, and rectangles, determine the diagonals of the square and rectangle.
17	3-4	Geometry	9	Shapes and objects, and their properties	34_09_17	Naming the sides and corners of triangles, squares, and rectangles, determining the side properties, and classifying the triangles according to their side lengths. Create structures suitable for drawings created with isometric or squared paper and identical cubes.
18	3-4	Geometry	11	Orientation in space and on a plane	34_11_01	Description of the relative position of objects in space or their representations, using appropriate geometric vocabulary (parallel, perpendicular, oblique, right, left, etc.). Verbal description and interpretation of movements, in relation to oneself or to other points of reference, using appropriate geometric vocabulary.

						Interpretation of itineraries in plans, using physical and virtual supports.
19	3-4	Geometry	11	Orientation in space and on a plane	34_11_02	Geometric models in solving problems related to the other senses. Recognition of geometric relationships in fields outside the mathematics class, such as art, science, and everyday life.
20	3-4	Geometry	11	Orientation in space and on a plane	34_11_03	Students practice using the concepts of direction and location.
21	3-4	Geometry	11	Orientation in space and on a plane	34_11_04	Properly uses terms describing directions and distances in two- and three-dimensions; is able to navigate their neighborhood and on a map.
22	3-4	Geometry	25	Planar and spatial shapes' constructions, transformations, properties, and classification	34_25_01	Recognize horizontal or vertical translations of an object by the number of cells. Recognize the rotation of an object around a point.
23	3-4	Geometry	25	Planar and spatial shapes' constructions, transformations, properties, and classification	34_25_02	Scale for enlargement and reduction, and the use of scale in student-centered situations.
24	3-4	Analysis	10	Constructions and Translations	34_10_01	This text discusses strategies and techniques for constructing two-dimensional geometric figures through composition and decomposition. It emphasizes the use of manipulative materials and drawing instruments, such as rulers and squares, as well as computer applications. It also explores the properties

						of two- and three-dimensional geometric figures using physical tools like grids, geoplanes, and polycubes, along with digital resources such as dynamic geometry programs, augmented reality, and educational robotics.
25	3-4	Analysis	10	Constructions and Translations	34_10_02	Identification of transformed figures by means of translations and symmetries in everyday life situations. Generation of transformed figures from symmetries and translations of an initial pattern and prediction of the result.
26	3-4	Analysis	10	Constructions and Translations	34_10_03	Guide the students to observe rotational and translational symmetry in their surroundings, for example, in art.
27	3-4	Analysis	10	Constructions and Translations	34_10_04	Creates a mirror image with movement, masking; creates symmetrical shapes; checks the correctness of a reflection, symmetrical line pattern
28	3-4	Measuring	6	Measurements and measurement tools	34_06_01	Comparison and ordering strategies of measures of the same magnitude (km, m, cm, mm; kg, g; l and ml): application of equivalences between units in everyday problems that involve converting to smaller units. Estimation of measures of length, mass, and capacity by comparison. Evaluation of the results of measurements, estimations, or calculations of measurements.
29	3-4	Measuring	6	Measurements and measurement tools	34_06_02	Measurable attributes of objects (length, mass, capacity, surface area, volume, and angle amplitude). Conventional (km, m, cm, mm; kg, g; l and ml) and non-conventional units in everyday situations. Measurement of time (year, month, week, day, hour, and minutes) and determination of the duration of time periods.
30	3-4	Measuring	6	Measurements and measurement tools	34_06_03	Strategies for measuring with non-conventional (repeating a unit, use of grids, and manipulative materials) and conventional instruments and units. Measurement processes using

						conventional instruments (ruler, tape measure, scales, analog and digital clocks).
31	3-4	Measuring	6	Measurements and measurement tools	34_06_04	Calculation and estimation of amounts and change (euros and cents) in everyday life problems: income, expenses, and savings. Responsible purchasing decisions.
32	3-4	Measuring	6	Measurements and measurement tools	34_06_05	Strategies for calculating the perimeters of plane figures and their use in solving everyday problems.
33	3-4	Measuring	6	Measurements and measurement tools	34_06_06	Practice measuring and pay attention to accuracy, result evaluation, and checking the measurement.
34	3-4	Measuring	6	Measurements and measurement tools	34_06_07	Measures the area and circumference of different polygons (possibly by coverage); approximates and measures using known units of measurement; knows the real-world size of units of measurement, and converts among units of measurement.
35	3-4	Measuring	6	Measurements and measurement tools	34_06_08	Explain the relationship between the perimeters of the square and the rectangle and the side lengths. Realize that the areas of the shapes are composed of unit squares covering these areas. On the other hand, there are gains in associating the area of a square and a rectangle with multiplication and addition operations.
36	3-4	Measuring	6	Measurements and measurement tools	34_06_09	Real-world decision problems using calculations (earnings, expenses, donations, savings...).
37	3-4	Measuring	6	Measurements and measurement tools	34_06_10	Upscale or downscale units of time measurement. Concepts of path and speed. Relationship between path, time, and speed.
38	3-4	Measuring	6	Measurements and measurement tools	34_06_11	Measuring area and circumference, length, our money, measuring time, weighing, and liquid. Know the millimeter and its relationship with other measurement units.
39	3-4	Measuring	6	Measurements and	34_06_12	Explain what the standard liquid measuring unit is and its necessity, and how to make

				measurement tools		measurements with liters and half liters. Measure by using liters and milliliters together, and to estimate the amount of liquid in a container with the measurement units they have learned.
40	3-4	Measuring	6	Measurements and measurement tools	34_06_13	Read the time in minutes and hours. Explain the relationship between year-week, year-day, minute-second without conversion operations. Hour-minute, minute-second, year-week, year-month-week-day relations and expressing one in terms of the other are discussed.
41	3-4	Measuring	6	Measurements and measurement tools	34_06_14	Notice and compare the relationship between money and coins, show this relationship, and solve the problems related to these relationships.
42	3-4	Measuring	6	Measurements and measurement tools	34_06_15	Recognition of where kilograms and grams are used and explanation of the relationship between these units. Estimate and investigate the accuracy after estimating the masses of the objects. Half and quarter kilograms are measured in grams, and kilograms and grams are used together when measuring mass. Achievements of estimating the places where tons.
43	3-4	Measuring	26	Measurements and Units	34_26_01	Guide the students to understand how the system of measurement units is structured. Practice unit conversions with the most commonly used measurement units.
44	3-4	Algebra	2	Problem solving	34_02_01	Compare different strategies to solve a problem in a patterned way. Obtain possible solutions to a problem following a known strategy. Demonstrate the mathematical correctness of the solutions to a problem and their coherence in the given context.
45	3-4	Algebra	2	Problem solving	34_02_02	A guided process of solving everyday problems.
46	3-4	Algebra	2	Problem solving	34_02_03	A structured process of modeling using mathematical representations (graphs,

						tables, etc.) to facilitate the understanding and resolution of everyday problems.
47	3-4	Algebra	2	Problem solving	34_02_04	Searches for a solution for problems; recalls their memory, re-states, solves problems; interprets and checks the solution; asks questions based on the problem, and illustrates the problem.
48	3-4	Algebra	2	Problem solving	34_02_05	Solve multi-step problems; up- or downscaling of units of measurement might be needed.
49	3-4	Algebra	2	Problem solving	34_02_06	Analyze simple mathematical conjectures by investigating patterns, properties, and relationships in a patterned way. Give examples of problems about everyday situations that are solved mathematically.
50	3-4	Algebra	2	Problem solving	34_02_07	Interpret, verbally or graphically, problems of daily life, understanding the questions posed through different strategies or tools, including technological ones. Produce mathematical representations through schemes or diagrams that help in the resolution of a problematized situation.
51	3-4	Algebra	2	Problem solving	34_02_08	Make connections between different mathematical elements, applying their own knowledge and experiences. Interpret situations in diverse contexts, recognizing connections between mathematics and everyday life.
52	3-4	Algebra	2	Problem solving	34_02_09	Recognize simple mathematical language present in everyday life in different formats, acquire basic specific vocabulary, and show understanding of the message. Explain mathematical processes and ideas, the steps followed in solving a problem, or the results obtained, using simple mathematical language in different formats.
53	3-4	Algebra	2	Problem solving	34_02_10	Get acquainted with the concept of the unknown (variable); distinguish between known and to be determined (unknown) data; and between relevant and irrelevant data; utilize a mathematical model, check the results, and compose an answer.

54	3-4	Algebra	2	Problem solving	34_02_11	Learn through examples: equation, unknown of an equation, solution of an equation. Use alternate equations to describe the same problem.
55	3-4	Algebra	2	Problem solving	34_02_12	Calculate the value of an expression with a letter in it, given the value of a letter. Form a letter expression from a word problem.
56	3-4	Algebra	2	Problem solving	34_02_13	Formulation of mathematical questions based on everyday situations.
57	3-4	Algebra	2	Problem solving	34_02_14	Unknown numbers and how they can be represented by a symbol.
58	3-4	Algebra	3	Comparison, sorting	34_03_01	Sorts based on own criteria, recognizes criteria in existing sorting, able to continue sorting
59	3-4	Algebra	3	Comparison, sorting	34_03_02	Compares finite sets based on the number of elements; pairs up elements of two sets with each other (1:1)
60	3-4	Algebra	3	Comparison, sorting	34_03_03	Understands and properly uses the more, less, equal amount relations, and the smaller, greater, equal relations with regard to numbers
61	3-4	Algebra	3	Comparison, sorting	34_03_04	Mathematical similarities and how the equals sign is used to draw simple equations.
62	3-4	Algebra	3	Comparison, sorting	34_03_05	Orders numbers and quantities by size; gives and understands numbers constructed through various operations; able to find the position of numbers on number lines and tables, recognizes numbers in their different forms up to 10,000
63	3-4	Algebra	3	Comparison, sorting	34_03_06	Sorts elements, inserts new elements
64	3-4	Algebra	4	Counting, approximations	34_04_01	Strategies and techniques for interpreting and manipulating the order of magnitude of numbers (tens, hundreds, and thousands). Reasoned estimates and approximations of quantities in problem-solving contexts. Uses tools (abacus), understands multiplication and division by 10, 100, 1000.

65	3-4	Algebra	4	Counting, approximations	34_04_02	Counts and measures using (arbitrarily chosen or standard) units for numbers up to 10,000; can count up and down by tens, hundreds, thousands; knows the following approximation methods: approximate counting, approximate measurement, measuring with a multiple of the measurement unit; knows how to refine their approximation by re-approximation.
70	3-4	Algebra	7	Equations, operations	34_07_01	Equality is an expression of an equivalence relationship between two elements and obtaining simple unknowns (represented by a symbol) in either element.
71	3-4	Algebra	7	Equations, operations	34_07_02	Representation of the relationship 'greater than' and 'less than', and use of the signs < and >.
72	3-4	Algebra	7	Equations, operations	34_07_03	Addition, subtraction, multiplication, and division of natural numbers are solved with flexibility and sense in contextualized situations: solving strategies, tools, and properties.
73	3-4	Algebra	7	Equations, operations	34_07_04	Natural numbers and fractions in everyday life contexts: comparison and ordering. Relationships between addition and subtraction, and multiplication and division: application in everyday contexts.
74	3-4	Algebra	7	Equations, operations	34_07_05	Equality and inequality relations, and use of “=” and “?” signs between expressions involving operations and their properties.
75	3-4	Algebra	7	Equations, operations	34_07_06	The understanding of the structure, connections, and divisibility of numbers is diversified by studying and classifying numbers.
76	3-4	Algebra	7	Equations, operations	34_07_07	Ensure that the students master the concept of multiplication. Learn multiplication tables 6-9. Ensure that the students master the multiplication tables 1-10.
77	3-4	Algebra	7	Equations, operations	34_07_08	Practice dividing into parts. (38/4=36/4+2/4)
78	3-4	Algebra	7	Equations, operations	34_07_09	Practice both partitive and quotative division.

79	3-4	Algebra	7	Equations, operations	34_07_10	Properly interprets the addition, subtraction, multiplication, and division for numbers up to 10,000; properly utilizes operations for a word problem, is able to use inverse operations; understands the following terms: addend, sum, minuend, subtrahend, difference, multiplicand, multiplier, product, dividend, divisor, quotient, remainder; knows the symbols for operations, uses parentheses for multiple operations.
80	3-4	Algebra	7	Equations, operations	34_07_11	How natural numbers and simple numbers as fractions are used in student-centered situations.
81	3-4	Algebra	7	Equations, operations	34_07_12	Properties of the four operations, their relationships, and their use in different situations.
82	3-4	Algebra	7	Equations, operations	34_07_13	Reading and writing 4, 5, and 6-digit numbers, dividing them into parts, and specifying the place values.
83	3-4	Algebra	8	Mental calculations	34_08_01	Mental calculation strategies with natural numbers and fractions. Strategies for recognizing which simple operations (addition, subtraction, multiplication, division as division and partition) are useful to solve contextualized situations. Construction of the multiplication tables based on the number of times, repeated addition, or grid arrangement.
84	3-4	Algebra	8	Mental calculations	34_08_02	Practice the basic operations, mental calculation (+, -, *, /).
85	3-4	Algebra	8	Mental calculations	34_08_03	Accurately adds and subtracts in the head for numbers up to 100, multiplies and divides.
86	3-4	Algebra	8	Mental calculations	34_08_04	Understands the relationship of multiplication and division tables.
87	3-4	Algebra	8	Mental calculations	34_08_05	Methods of calculating using natural numbers when calculating mental arithmetic and approximate estimates, and written calculation. Use of digital tools in calculations.
88	3-4	Algebra	8	Mental calculations	34_08_06	Reinforcing the mental multiplication and division operations.

89	3-4	Algebra	12	Relationships	34_12_01	Students deepen their skills in comparing, classifying, and ordering, searching for answer options systematically, and observing cause and effect relationships in maths.
90	3-4	Algebra	12	Relationships	34_12_02	Finds pairs in a memory game; recognizes and expresses relationships; looks for patterns among elements of a series.
91	3-4	Algebra	12	Relationships	34_12_03	Creates a series based on a given rule; lists months; recognizes relationships among element doubles or triples.
92	3-4	Algebra	12	Relationships	34_12_04	Extends, describes sequences of 2-4 repeating members. Differences can be in size, color, line thickness, angle of rotation, and sequence might carry over to the next line. Investigate sequences obtained by merging two sequences.
93	3-4	Algebra	12	Relationships	34_12_05	Simple patterns in number sequences and simple geometric patterns, and how they are constructed, described, and expressed.
94	3-4	Algebra	12	Relationships	34_12_06	Simple tables and diagrams are used to categorize data and describe results from investigations, both with and without digital tools.
95	3-4	Algebra	15	Columnar operations	34_15_01	Practice addition and subtraction algorithms, ensuring that the skill is learned. (columnar addition and subtraction)
97	3-4	Algebra	15	Columnar operations	34_15_02	Practice the multiplication algorithm and ensure that the skill is mastered (columnar multiplication with one- and two-digit multipliers).
99	3-4	Algebra	15	Columnar operations	34_15_03	Interprets and checks the solution of columnar multiplication with one- and two-digit multipliers and division with one-digit divisors; approximates.
100	3-4	Algebra	16	Part-Part-Whole Relationships	34_16_01	Proper fractions with a denominator up to 12 in everyday contexts.
101	3-4	Algebra	16	Part-Part-Whole Relationships	34_16_02	Learn the concept of fractions and practice basic calculations of fractions in different situations.

102	3-4	Algebra	16	Part-Part-Whole Relationships	34_16_03	Illustrates, draws, measures, and understands the unit fractions and their multiples
103	3-4	Algebra	16	Part-Part-Whole Relationships	34_16_04	Compare fractions m/n < 1 where numerators or denominators are the same. Add and subtract decimal numbers with 1 or 2 decimal places
104	3-4	Algebra	16	Part-Part-Whole Relationships	34_16_05	Fractions as part of a whole and part of a number, and how parts are named and expressed as simple fractions. How simple fractions are related to natural numbers.
105	3-4	Algebra	16	Part-Part-Whole Relationships	34_16_06	Division (grouping, segmentation) process, fractionally introduced by emphasizing the part-whole relationship. The concept of a unit fraction and the relationship between the numerator and the denominator is reinforced.
106	3-4	Algebra	16	Part-Part-Whole Relationships	34_16_07	Define and use simple, compound, and integer fractions, and addition and subtraction of fractions. Add and subtract fractions with equal denominators and solve appropriate problems.
107	3-4	Algebra	17	Natural numbers, Integers, and their operations	34_17_01	Directed units (temperature); understands lesser and greater relations for negative numbers, too.
108	3-4	Algebra	17	Natural numbers, Integers, and their operations	34_17_02	Students are guided to round numbers and calculate with approximations so that they learn to estimate the order of magnitude of the result.
109	3-4	Algebra	17	Natural numbers, Integers, and their operations	34_17_03	The four basic arithmetic operations (addition, subtraction, multiplication, and division) and rules for their use in calculations with natural numbers.
110	3-4	Algebra	31	Pattern usage/recognition	34_31_01	Verbal description from regularities in a collection of numbers, figures, or pictures.
111	3-4	Algebra	31	Pattern usage/recognition	34_31_02	Identification, verbal description, representation, and reasoned prediction of

						terms from regularities in a collection of numbers, figures, or pictures.
112	3-4	Number system	5	Digits, numbers, number systems	34_05_01	Identifies, tells apart, and describes objects, things, and people based on properties.
113	3-4	Number system	5	Digits, numbers, number systems	34_05_02	Recognition of similar and differing attributes, organizing into groups (find similarities, differences).
114	3-4	Number system	5	Digits, numbers, number systems	34_05_03	Identifies and selects elements matching multiple criteria (find similarities, regularities).
115	3-4	Number system	5	Digits, numbers, number systems	34_05_04	Describes the properties of numbers; describes and with relation to other numbers; knows and recognizes Roman numerals.
116	3-4	Number system	5	Digits, numbers, number systems	34_05_05	Natural numbers and their properties and how numbers can be divided, and how they can be used to specify quantities and order.
117	3-4	Number system	5	Digits, numbers, number systems	34_05_06	Symbols for numbers and the historical development of symbols in some different cultures through history.
118	3-4	Number system	5	Digits, numbers, number systems	34_05_07	Reading, representation (including the number line and with manipulative materials), composition, decomposition, and re-composition of natural numbers up to 9999.
119	3-4	Number system	5	Digits, numbers, number systems	34_05_08	Base ten number system (up to 9999): application of the relations it generates in operations.
120	3-4	Number system	5	Digits, numbers, number systems	34_05_09	Deepen and enforce students' perception of the decimal numeral system (place-value).
121	3-4	Number system	5	Digits, numbers, number systems	34_05_10	Understands how number systems work; breaks up numbers into sums based on powers of 10; understands place and face value.

122	3-4	Number system	5	Digits, numbers, number systems	34_05_11	The positional number system and how it is used to describe natural numbers.
123	3-4	Number system	5	Digits, numbers, number systems	34_05_12	Model and analyze three-digit numbers and thus expand and reinforce the knowledge of place value. Introduction of the number systems and numbers used by ancient civilizations.
124	3-4	Number system	5	Digits, numbers, number systems	34_05_13	Different number systems have been used in various cultures throughout history.
125	3-4	Mathematics, logic/set theory	1	Categorization, Classification	34_01_01	Regularities: Creates statements about a given set; uses the terms 'all', 'not all', 'exists...', 'none of ...', and their synonyms appropriately
126	3-4	Mathematics, logic/set theory	1	Categorization, Classification	34_01_02	Students improve their skills in finding similarities, differences, and regularities.
127	3-4	Mathematics, logic/set theory	1	Categorization, Classification	34_01_03	Recognition of similar and differing attributes, organizing into groups, and creating a Venn diagram
128	3-4	Mathematics, logic/set theory	1	Categorization, Classification	34_01_04	Organizes into sets, takes two criteria into consideration at the same time, and names the organizing criteria
129	3-4	Mathematics, logic/set theory	1	Categorization, Classification	34_01_05	Identifies and selects elements matching multiple criteria
130	3-4	Mathematics, logic/set theory	19	Math and logic in everyday life	34_19_01	Determines "true" and "false" statements; creates statements with the terms 'all', 'not all', 'exists...', 'none of ...', and their synonyms appropriately
131	3-4	Probability and Statistics	13	Data collection and management	34_13_01	Statistical graphs of everyday life (pictograms, bar charts, histograms...): reading, interpretation. Simple strategies for the collection, classification, and organization of discrete qualitative or quantitative data in small samples using a calculator and simple computer applications. Absolute frequency: interpretation.

132	3-4	Probability and Statistics	13	Data collection and management	34_13_02	Simple statistical graphs to represent data, selecting the most convenient, using traditional resources and simple computer applications. Mode: interpretation as the most frequent data. Graphical comparison of two sets of data to establish relationships and draw conclusions.
133	3-4	Probability and Statistics	13	Data collection and management	34_13_03	Collects data in their environment; records data for later evaluation; organizes collected data in a table, and illustrates it on a diagram.
134	3-4	Probability and Statistics	13	Data collection and management	34_13_04	Read and interpret simple tables with at most three data groups, and organize the data obtained from the table. Examine and create the column chart. Use different representations to present the data, solve and set up problems related to daily life by using the information shown in tree diagrams, column graphs, tables, and other graphics.
135	3-4	Probability and Statistics	14	Randomness	34_14_01	Formulation of conjectures from data collected and analyzed, making sense of them in the context of the study.
136	3-4	Probability and Statistics	14	Randomness	34_14_02	Probability as a subjective measure of uncertainty. Recognition of uncertainty in everyday situations and by performing experiments. Identification of certain events, possible events, and impossible events. Comparing the probability of two events intuitively.
137	3-4	Probability and Statistics	14	Randomness	34_14_03	Random events in specific situations.
138	3-4	Probability and Statistics	21	Ratios, percentages	34_21_01	Proportional relationships, including doubling and halving.
139	5-6	Geometry	9	Shapes and objects, and their properties	56_09_01	Classify 2D shapes into polygons and others, and study their properties.
140	5-6	Geometry	9	Shapes and objects, and	56_09_02	Learn about the concepts of point, segment, line, and angle.

				their properties		
141	5-6	Geometry	9	Shapes and objects, and their properties	56_09_03	Find similarities, differences, and regularities.
142	5-6	Geometry	11	Orientation in space and on a plane	56_11_01	Guide the students to use the scale when using the map.
143	5-6	Geometry	11	Orientation in space and on a plane	56_11_02	Basic two- and three-dimensional geometrical objects and their properties and relationships. Construction of geometrical objects.
144	5-6	Geometry	25	Planar and spatial shapes' constructions, transformations, properties, and classification	56_25_01	Geometric figures in everyday objects: identification and classification according to their elements and the relationships between them. Geometric vocabulary: verbal description of the elements and properties of geometric figures.
145	5-6	Geometry	25	Planar and spatial shapes' constructions, transformations, properties, and classification	56_25_02	Measure and calculate the perimeters and areas of different shapes and the volumes of rectangular cuboids.
146	5-6	Geometry	25	Planar and spatial shapes' constructions, transformations, properties, and classification	56_25_03	Learn more about triangles, quadrilaterals, and circles. Group triangles based on their angles and sides.

147	5-6	Geometry	25	Planar and spatial shapes' constructions, transformations, properties, and classification	56_25_04	Methods for determining and estimating the circumference and areas of different two-dimensional geometrical figures.
148	5-6	Geometry	25	Planar and spatial shapes' constructions, transformations, properties, and classification	56_25_05	Explain, show, and draw basic geometric concepts such as a line, a line segment, and a ray. Name polygons and recognize their basic elements of rectangle, parallelogram, rhombus, and trapezoid.
149	5-6	Geometry	25	Planar and spatial shapes' constructions, transformations, properties, and classification	56_25_06	Construction techniques of geometric figures by composition and decomposition, using manipulative materials, drawing instruments, and computer applications.
150	5-6	Geometry	25	Planar and spatial shapes' constructions, transformations, properties, and classification	56_25_07	Location and movements on plans and maps from reference points (including cardinal points), directions, and calculation of distances (scales): description and interpretation with the appropriate vocabulary in physical and virtual supports.
151	5-6	Geometry	25	Planar and spatial shapes'	56_25_08	Transformations by means of rotations, translations, and symmetries in everyday life situations: identification of

				constructions, transformations, properties, and classification		transformed figures, generation from initial patterns, and prediction of the result. Similarity in everyday life situations: identification of similar figures, generation from initial patterns, and prediction of the result.
152	5-6	Geometry	25	Planar and spatial shapes' constructions, transformations, properties, and classification	56_25_09	Strategies for calculating areas and perimeters of plane figures in everyday life situations. Geometric models in solving problems related to the other senses. Elaboration of conjectures about geometric properties, using drawing instruments (compass and protractor) and dynamic geometry programs. Geometric ideas and relationships in art, science, and everyday life.
153	5-6	Geometry	25	Planar and spatial shapes' constructions, transformations, properties, and classification	56_25_10	Learn about the concept of scale and use it in enlargements and reductions. Gain practical experience in movement along a plane. Know the basic constructions: creating a perpendicular bisector, angle bisector, parallel and perpendicular lines, copying an angle.
154	5-6	Geometry	25	Planar and spatial shapes' constructions, transformations, properties, and classification	56_25_11	Scale for enlargement and reduction, and the use of scale in student-centered situations.
155	5-6	Geometry	25	Planar and spatial shapes' constructions, transformati	56_25_12	Symmetry in plane and how symmetry can be constructed.

				ons, properties, and classification		
156	5-6	Geometry	25	Planar and spatial shapes' constructions, transformations, properties, and classification	56_25_13	Students take a closer look at a rectangular cone, a circular cylinder, a circular cone, and a pyramid.
157	5-6	Geometry	25	Planar and spatial shapes' constructions, transformations, properties, and classification	56_25_14	Comparing, estimating, and measuring length, area, mass, volume, time, and angles using common units of measurement, including unit conversions related to them
158	5-6	Analysis	10	Constructions and Translations	56_10_01	Statistical data sets and graphs of everyday life. Strategies for conducting a simple statistical study. Absolute and relative frequency tables.
159	5-6	Analysis	10	Constructions and Translations	56_10_02	Simple statistical graphs (bar chart, pie chart, histogram, etc.). Measures of centralization. Measures of dispersion (range).
160	5-6	Analysis	10	Constructions and Translations	56_10_03	Calculator and other digital resources, such as a spreadsheet, to organize statistical information and perform different data visualizations. Relation and comparison of two sets of data from their graphical representation.
161	5-6	Analysis	23	Functional relationships	56_23_01	Description of positions and movements in the first quadrant of the Cartesian coordinate system.

162	5-6	Analysis	23	Functional relationships	56_23_02	Learn about the first quadrant of the coordinate system and extend then to all quadrants.
163	5-6	Analysis	23	Functional relationships	56_23_03	Finds their way around a coordinate system.
164	5-6	Analysis	23	Functional relationships	56_23_04	Coordinate system and grading of coordinate axes.
165	5-6	Measuring	6	Measurements and measurement tools	56_06_01	Practice measuring and pay attention to accuracy, result evaluation, and checking the measurement.
166	5-6	Measuring	6	Measurements and measurement tools	56_06_02	Solving problems related to responsible consumption.
167	5-6	Measuring	6	Measurements and measurement tools	56_06_03	Guide the students to use the scale when using the map.
168	5-6	Measuring	6	Measurements and measurement tools	56_06_04	Solve movement problems using diagrams and various models.
169	5-6	Measuring	26	Measurements and Units	56_26_01	Instruments (analog or digital) and appropriate units to measure lengths, objects, angles, and times.
170	5-6	Measuring	26	Measurements and Units	56_26_02	Measure and calculate the perimeters and areas.
171	5-6	Algebra	3	Comparison, sorting	56_03_01	Students deepen their skills in comparing, classifying, and ordering, searching for answer options systematically, and observing cause and effect relationships in maths.
172	5-6	Algebra	17	Natural numbers, Integers, and their operations	56_17_01	Ground the concept of a negative number and expand the number range with negative integers.
173	5-6	Algebra	17	Natural numbers, Integers, and their operations	56_17_02	Varied counting strategies, systematic counting, and adapting counting to the size of numbers.

174	5-6	Algebra	17	Natural numbers, Integers, and their operations	56_17_03	Strategies and techniques for interpreting and manipulating the order of magnitude of numbers.
175	5-6	Algebra	17	Natural numbers, Integers, and their operations	56_17_04	Guided to round numbers and calculate with approximations, learn to estimate the order of magnitude of the result.
176	5-6	Algebra	17	Natural numbers, Integers, and their operations	56_17_05	When solving practical problems, divide by at most 2-digit numbers in writing. Approximates the quotient.
177	5-6	Algebra	17	Natural numbers, Integers, and their operations	56_17_06	Compare and round natural numbers using different methods. Define a coordinate plane, and understand how pairs of numbers represent a point on it.
178	5-6	Algebra	17	Natural numbers, Integers, and their operations	56_17_07	Perform four operations on natural numbers and integers.
179	5-6	Algebra	17	Natural numbers, Integers, and their operations	56_17_08	Determines the given numbers' negative, absolute value; knows integers.
180	5-6	Algebra	20	Rational numbers, fractions, and their operations	56_20_01	Reading, representation, composition, decomposition, and re-composition of natural numbers, decimals to thousandths, fractions, and decimals to express quantities, and choosing the best representation for each situation or problem.
181	5-6	Algebra	20	Rational numbers, fractions, and their operations	56_20_02	Base ten numbering system (natural numbers and decimals to thousandths): application of the relations it generates in operations.

182	5-6	Algebra	20	Rational numbers, fractions, and their operations	56_20_03	Familiarize themselves with decimal numbers as part of the decimal system and practice basic calculations with decimal numbers.
183	5-6	Algebra	20	Rational numbers, fractions, and their operations	56_20_04	Knows and utilizes place value notation for decimal fractions, fractions, and decimals to thousandths.
184	5-6	Algebra	20	Rational numbers, fractions, and their operations	56_20_05	Rational numbers, including negative numbers, and their properties, and how the numbers can be divided and used.
185	5-6	Algebra	20	Rational numbers, fractions, and their operations	56_20_06	The positional number system and how it is used to describe whole numbers and decimal numbers.
186	5-6	Algebra	20	Rational numbers, fractions, and their operations	56_20_07	Methods for calculations with natural numbers, simple fractions, and decimals in approximate estimates, mental arithmetic, and written calculations using digital tools.
187	5-6	Algebra	20	Rational numbers, fractions, and their operations	56_20_08	Mental calculation strategies with natural numbers, fractions, and decimals. Strategies for recognizing which simple or combined operations (+, -, *, /) are useful to solve contextualized situations.
188	5-6	Algebra	20	Rational numbers, fractions, and their operations	56_20_09	Understand that operations with decimal numbers are similar to those with whole numbers. Additionally, addition, subtraction, multiplication, and division with rational numbers are visualized and justified.
189	5-6	Algebra	20	Rational numbers, fractions, and their operations	56_20_10	How numbers in fractions and decimals can be used in everyday situations.
190	5-6	Algebra	22	Problem-solving with	56_22_01	Equality and inequality relations and use of the signs < and >. Determination of

				equations, proofing		unknown data (represented by a letter or symbol) in simple expressions related by means of these signs and the signs = and ?.
191	5-6	Algebra	22	Problem-solving with equations, proofing	56_22_02	Study equations and find solutions by reasoning and experimenting. Interprets and double-checks the result. First-degree equations are solved. Real-world problems with direct proportionality are discussed. Ratio and proportionality are defined. Properties of proportions are understood and used to solve problems.
192	5-6	Algebra	22	Problem-solving with equations, proofing	56_22_03	Formulation of mathematical questions based on everyday situations. Strategies for solving mathematical problems in student-related situations. Formulate simple mathematical conjectures by investigating patterns, properties, and relationships in a guided manner. Pose new problems that are solved mathematically.
193	5-6	Algebra	22	Problem-solving with equations, proofing	56_22_04	Use connections between different mathematical elements by mobilizing one's knowledge and experiences. Use connections between mathematics. The process of modeling everyday problems using mathematical representations.
194	5-6	Algebra	22	Problem-solving with equations, proofing	56_22_05	Select among different strategies to solve a problem, justifying the choice. Obtain possible solutions to a problem, selecting among several known strategies in an autonomous way. Verify the mathematical correctness of the solutions to a problem and their coherence in the given context. Solves word problems through deduction or equations; approximates.
195	5-6	Algebra	22	Problem-solving with equations, proofing	56_22_06	Create and re-arrange simple alphabetic expressions using natural numbers. Methods, including algebraic, for solving simple equations.
196	5-6	Algebra	22	Problem-solving with equations, proofing	56_22_07	Linear and quadratic relationships in everyday or mathematically relevant situations: expression using symbolic algebra. Formulate and check simple conjectures in a guided way, analyzing

						patterns, properties, and relationships. Propose variants of a given problem by modifying some of its data or some condition of the problem. Use appropriate technological tools in the investigation and verification of conjectures or problems.
197	5-6	Algebra	22	Problem-solving with equations, proofing	56_22_08	Recognize situations that can be formulated and solved using mathematical tools and strategies, establishing connections between the real world and mathematics, and using the processes inherent to research: inferring, measuring, communicating, classifying, and predicting. Identify coherent connections between mathematics and other subjects by solving contextualized problems. Interpret mathematical problems by organizing data, establishing relationships between them, and understanding the questions asked. Apply appropriate tools and strategies.
198	5-6	Algebra	22	Problem-solving with equations, proofing	56_22_09	Obtain mathematical solutions to a problem, activating knowledge and using the necessary technological tools. Recognize the relationships between mathematical knowledge and experiences, forming a coherent whole. Make connections between different mathematical processes by applying prior knowledge and experiences. Modelling of everyday situations using mathematical representations and algebraic language.
199	5-6	Algebra	22	Problem-solving with equations, proofing	56_22_10	Represent mathematical concepts, procedures, information, and results in different ways and with different tools, including digital tools, visualizing ideas, structuring mathematical processes, and valuing their usefulness for sharing information. Elaborate mathematical representations that help in the search for strategies to solve a problematized situation. Equivalence of algebraic

						expressions in solving problems based on linear and quadratic relationships.
200	5-6	Algebra	22	Problem-solving with equations, proofing	56_22_11	Form and solve first-order equations and incomplete quadratic equations. Learn about the concept of a variable and practice addition, subtraction, and multiplication of polynomials. With simple lettered expressions, calculate addition, subtraction, and substitution value. Solves one variable, first-degree equations using different methods. Defining the concepts of unary, binary, ternary, and polynomial. Multiplying alphabetic expressions.
201	5-6	Algebra	22	Problem-solving with equations, proofing	56_22_12	Solve problems that require you to select solutions to inequalities that satisfy certain conditions. Dealing with various real-world situations that can be modeled by systems of equations.
202	5-6	Algebra	24	Series (Sequences)	56_24_01	Studying the regularity of number sequences and continuing number sequences according to a rule.
203	5-6	Algebra	24	Series (Sequences)	56_24_02	Continues periodic series based on a given rule; recognizes and describes the generating rule of a series given by a few of its members.
204	5-6	Algebra	24	Series (Sequences)	56_24_03	Solve problems in a variety of contexts where different ways of describing number sequences are considered, applied, and combined. Problem situations are addressed by identifying gaps in mathematical information and learning how to find and retrieve it.
205	5-6	Algebra	30	Number theory, LCM, GCD, power, root	56_30_01	Relationships between arithmetic operations: application in everyday contexts. Relationship of divisibility: multiples and divisors.
206	5-6	Algebra	30	Number theory, LCM, GCD, power, root	56_30_02	Students familiarize themselves with the divisibility of numbers and divide numbers into prime factors. knows and uses the divisibility rules for 2, 3, 4, 5, 6, 9, 10, 100; groups numbers based on the number of divisors or remainders

207	5-6	Algebra	30	Number theory, LCM, GCD, power, root	56_30_03	Power is a product of equal factors. Squares and cubes.
209	5-6	Algebra	31	Pattern usage/recognition	56_31_01	Creation of recurring patterns from regularities or other patterns using numbers, figures, or images.
210	5-6	Algebra	31	Pattern usage/recognition	56_31_02	Strategies for identification, oral description, discovery of hidden elements, and extension of sequences from regularities in a collection of numbers, figures, or images.
211	5-6	Algebra	31	Pattern usage/recognition	56_31_03	Recognize patterns facilitating its computational interpretation.
212	5-6	Number system	5	Digits, numbers, number systems	56_05_01	Understands and uses the place value notation of large numbers;
213	5-6	Number system	5	Digits, numbers, number systems	56_05_02	Different number systems have been used in various cultures throughout history.
214	5-6	Number system	5	Digits, numbers, number systems	56_05_03	Read and write natural numbers
215	5-6	Mathematics, logic/set theory	18	Sets	56_18_01	Recognize sets in concrete cases
216	5-6	Mathematics, logic/set theory	18	Sets	56_18_02	Illustrate sets in concrete cases
217	5-6	Mathematics, logic/set theory	19	Math and logic in everyday life	56_19_01	Understand problems of everyday life and elaborate mathematical representations to aid problem-solving.
218	5-6	Mathematics, logic/set theory	19	Math and logic in everyday life	56_19_02	Interpret simple mathematical language in various formats, acquire appropriate vocabulary, and effectively communicate mathematical ideas.
219	5-6	Mathematics, logic/set theory	19	Math and logic in everyday life	56_19_03	Practice activities that require logical thinking, including identifying rules and

						dependencies, and determining the number of options in math problems.
220	5-6	Mathematics, logic/set theory	19	Math and logic in everyday life	56_19_04	Strengthen students' skills in reasoning and justification.
221	5-6	Mathematics, logic/set theory	19	Math and logic in everyday life	56_19_05	Determine the logical value (true or false) of statements and understand various methods of justification, including mathematical proof.
222	5-6	Probability and Statistics	13	Data collection and management	56_13_01	Students' skills to systematically collect information on interesting topics are developed.
223	5-6	Probability and Statistics	13	Data collection and management	56_13_02	Create and interpret simple tables and bar graphs.
224	5-6	Probability and Statistics	13	Data collection and management	56_13_03	Students store and present information using tables and diagrams.
225	5-6	Probability and Statistics	14	Randomness	56_14_01	Students familiarize themselves with probability based on everyday situations by concluding whether an event is impossible, possible, or certain.
226	5-6	Probability and Statistics	14	Randomness	56_14_02	Randomness in games: understanding "certain", "Impossible", "possible but not certain"
227	5-6	Probability and Statistics	14	Randomness	56_14_03	Approximating the probability of events, counting events, and comparing the approximation with the result (for example, in a game)
228	5-6	Probability and Statistics	14	Randomness	56_14_04	Handling charts with large amounts of data. Probabilistic games were discussed and developed, where all players have the same chance of winning. Drawing charts and data tables, finding numerical characteristics, digital technologies are used.
229	5-6	Probability and Statistics	21	Ratios, percentages	56_21_01	Strategies for comparing and ordering measurements of the same magnitude, applying the equivalences between units. Evaluation of results of measurements and

						estimations or calculations of measurements, reasoning.
230	5-6	Probability and Statistics	21	Ratios, percentages	56_21_02	Conventional units of the Decimal Metric System (length, mass, capacity, volume, and surface area), time, and degree (angles) in everyday life contexts: selection and use of appropriate units. Instruments (analog or digital) and appropriate units to measure lengths, objects, angles, and times.
231	5-6	Probability and Statistics	21	Ratios, percentages	56_21_03	Solving problems related to responsible consumption and money. Numerical information in simple financial contexts. Methods for making responsible consumption decisions.
232	5-6	Probability and Statistics	21	Ratios, percentages	56_21_04	Proportional and non-proportional situations in everyday life problems. Solving problems of proportionality, percentages, and scales through the equality between ratios, reduction to unity, or the use of proportionality coefficients. Percentages greater than 100 and less than 1.
233	5-6	Probability and Statistics	21	Ratios, percentages	56_21_05	Relationship between simple fractions, decimals, and percentages. Comparison and ordering of fractions, decimals, and percentages: exact or approximate location on the number line.
234	5-6	Probability and Statistics	21	Ratios, percentages	56_21_06	Learn how to calculate the changed value, basic value, and change and comparison percentage.
235	5-6	Probability and Statistics	21	Ratios, percentages	56_21_07	Understands the concept of percentage; solves economic, finance, and everyday life problems related to percentages. Percentage is used in real-world problems about buying, selling, and discounts. Financial decisions are based on calculations.
236	5-6	Probability and Statistics	21	Ratios, percentages	56_21_08	Examines input/output (I and/or O) tables expressing the inverse proportionality relation, learning how to construct such tables and relate them to the problem condition. Solve problems in a variety of

						contexts where quantities are related by a linear relationship.
237	5-6	Probability and Statistics	21	Ratios, percentages	56_21_09	Graphs for expressing proportional relationships. Numbers in percentage form and their relation to numbers in fraction and decimal form. Proportional relationships, including doubling and halving. Proportionality and how proportional relationships are expressed in fraction, decimal, and percent form. Proportionality and how it is used to express scale, uniformity, and change. Percentage and change factor to express change, as well as calculations with percentages, in everyday situations and in different subject areas.
238	5-6	Probability and Statistics	27	Descriptive statistics	56_27_01	Identification of a data set as a sample of a larger set and reflection on the population to which it is possible to apply the conclusions of simple statistical investigations.
239	5-6	Probability and Statistics	27	Descriptive statistics	56_27_02	Formulation of appropriate questions that allow for knowing the characteristics of interest of a population. Relevant data to answer questions posed in statistical investigations. Strategies for drawing conclusions from a sample to make judgments and appropriate decisions.
240	5-6	Probability and Statistics	27	Descriptive statistics	56_27_03	Statistical data sets and graphs of everyday life: description, interpretation, and critical analysis. Strategies for conducting a simple statistical study: formulation of questions, and collection, recording, and organization of qualitative and quantitative data from different experiments (surveys, measurements, observations...). Absolute and relative frequency tables: interpretation.
241	5-6	Probability and Statistics	27	Descriptive statistics	56_27_04	Simple statistical graphs. Measures of centralization (mean and mode). Measures of dispersion (range).

242	5-6	Probability and Statistics	27	Descriptive statistics	56_27_05	Calculator and other digital resources, such as spreadsheets, to organize statistical information and perform different data visualizations. Relation and comparison of two sets of data from their graphical representation: formulation of conjectures, analysis of dispersion, and drawing conclusions.
243	5-6	Probability and Statistics	27	Descriptive statistics	56_27_06	Strategies for collecting and organizing data from daily life situations involving a single variable. Difference between a variable and single values. Analysis and interpretation of statistical tables and graphs of qualitative, discrete quantitative, and continuous quantitative variables in real contexts. Statistical graphs: representation using different technologies (calculator, spreadsheet, applications...) and choice of the most appropriate one.
244	5-6	Probability and Statistics	27	Descriptive statistics	56_27_07	Location measures: interpretation and calculation with technological support in real situations. Variability: interpretation and calculation, with technological support, of dispersion measures in real situations. Comparison of two data sets according to location and dispersion measures.
245	5-6	Probability and Statistics	27	Descriptive statistics	56_27_08	Ensure understanding of the average value and type value.
246	5-6	Probability and Statistics	27	Descriptive statistics	56_27_09	Deepen students' skills in collecting, structuring, and analyzing information.
247	5-6	Probability and Statistics	27	Descriptive statistics	56_27_10	Practice determining frequency, relative frequency, and median.
248	5-6	Probability and Statistics	27	Descriptive statistics	56_27_11	Use proportion to solve problems.
249	5-6	Probability and Statistics	27	Descriptive statistics	56_27_12	Statistics: maximum and minimum, average, mode, and median.

250	5-6	Probability and Statistics	28	Probability theory	56_28_01	Uncertainty in everyday life situations. Calculation of probabilities in experiments, comparisons, or investigations in which Laplace's rule is applicable. Deterministic and random phenomena: identification. Simple experiments.
251	5-7	Probability and Statistics	28	Probability theory	56_28_02	Assignment of probabilities by experimentation, the concept of relative frequency, and Laplace's rule. Calculate the probabilities. Construct two-outcome feasibility/probability trees and tables.
252	5-8	Probability and Statistics	28	Probability theory	56_28_03	Perform simple combinatorial analysis in concrete situations. Plays probabilistic games, performs probabilistic experiments, and through these collects, organizes, and displays the data according to a plan, also digitally.
253	5-9	Probability and Statistics	28	Probability theory	56_28_04	Various diagrams are interpreted and created. Explaining how cumulative frequency and cumulative relative frequency table data are represented by a cumulative frequency or cumulative relative frequency chart, and how to read and interpret the data represented by such charts.
254	5-10	Probability and Statistics	28	Probability theory	56_28_05	Understands the concepts of frequency and relative frequency. Uses this knowledge to explain 'impossible', 'certain', and 'less/more likely' statements.
255	5-11	Probability and Statistics	28	Probability theory	56_28_06	Get acquainted with the concept of standard deviation. Understand measures of central tendency and measures of dispersion, and how they are used for assessing results of statistical studies.
256	5-12	Probability and Statistics	28	Probability theory	56_28_07	Explaining the nature of different types of data and how variability in datasets can be interpreted in practice.
257	5-13	Probability and Statistics	28	Probability theory	56_28_08	Explore random events, chance, and risk based on observations, simulations, and statistical data. Compare the probabilities in different random trials.

262	7-8	Geometry	25	Planar and spatial shapes' constructions, transformations, properties, and classification	78_25_01	Planar representations of three-dimensional objects are used in the visualization and resolution of area problems.
263	7-8	Geometry	25	Planar and spatial shapes' constructions, transformations, properties, and classification	78_25_02	Plane and three-dimensional geometric figures: description and classification according to their properties or characteristics.
264	7-8	Geometry	25	Planar and spatial shapes' constructions, transformations, properties, and classification	78_25_03	Know and use the Pythagorean theorem to solve problems; know the parts of circles, differentiate between line, half-line, and section. Calculate the length of the circle and the segment of the circle, and the area of the circle and the circle slice, by evaluating the arcs in which the central angle is seen in the circle. Understand the Pythagorean relation and solve related problems.
265	7-8	Geometry	25	Planar and spatial shapes' constructions, transformations, properties, and classification	78_25_04	Students expand their understanding of the concepts of point, segment, straight line, and angle, and familiarize themselves with the concepts of line and ray. Properties related to straight lines, angles, and polygons are studied.
266	7-8	Geometry	25	Planar and spatial shapes'	78_25_05	Calculate the perimeters and areas of polygons. Know the properties of quadrilaterals: the sum of interior and

				constructions, transformations, properties, and classification		exterior angles, the differences between convex and concave, and the concept of a diagonal. Know the special quadrilaterals: trapezoid, parallelogram, rectangle, kite, rhombus, isosceles trapezoid, square; use their properties to solve problems.
267	7-8	Geometry	25	Planar and spatial shapes' constructions, transformations, properties, and classification	78_25_06	Geometric relations such as congruence, similarity, and the Pythagorean relation in plane and three-dimensional figures: identification and application.
268	7-8	Geometry	25	Planar and spatial shapes' constructions, transformations, properties, and classification	78_25_07	Translation and reflection transformations.
269	7-8	Geometry	25	Planar and spatial shapes' constructions, transformations, properties, and classification	78_25_08	Construction of geometric figures with manipulative and digital tools (dynamic geometry programs, augmented reality).
270	7-8	Geometry	25	Planar and spatial shapes' constructions, transformati	78_25_09	Elementary transformations such as rotations, translations, and symmetries in various situations using technological or manipulative tools.

				ons, properties, and classification		
271	7-8	Geometry	25	Planar and spatial shapes' constructions, transformations, properties, and classification	78_25_10	Geometric modeling: numerical and algebraic relationships in problem-solving. Geometric relations in mathematical and non-mathematical contexts (art, science, daily life).
272	7-8	Geometry	25	Planar and spatial shapes' constructions, transformations, properties, and classification	78_25_11	Practice geometric construction. Strengthen the understanding of the concepts of similarity and congruence.
273	7-8	Geometry	25	Planar and spatial shapes' constructions, transformations, properties, and classification	78_25_12	Learn to prove the congruency of shapes in the coordinate plane by showing the sequence of transformations to get from one shape to another.
274	7-8	Geometry	25	Planar and spatial shapes' constructions, transformations, properties,	78_25_13	Learn how to formulate the converse of a conditional statement. Through case studies, it is shown that not every inverse statement is true. Address a variety of mathematical and practical problems involving the combination of existing knowledge of shapes with knowledge of other areas.

				and classification		
275	7-8	Geometry	25	Planar and spatial shapes' constructions, transformations, properties, and classification	78_25_14	Geometrical objects and their properties and relationships. Construction of geometrical objects, both with and without digital tools.
276	7-8	Geometry	25	Planar and spatial shapes' constructions, transformations, properties, and classification	78_25_15	Geometrical theorems, formulae, and arguments for their validity.
277	7-8	Geometry	25	Planar and spatial shapes' constructions, transformations, properties, and classification	78_25_16	Mathematical similarities and how the equal sign is used to draw equations and functions.
278	7-8	Geometry	25	Planar and spatial shapes' constructions, transformations, properties, and classification	78_25_17	Scale for reduction and enlargement of two- and three-dimensional objects. Uniformity and congruence.

279	7-8	Geometry	25	Planar and spatial shapes' constructions, transformations, properties, and classification	78_25_18	The concepts of bisector, corresponding, inverse, interior reverse, and exterior reverse angles are discussed, and their properties are examined. The concepts of congruence and similarity in polygons are examined, identifying and constructing congruent and similar polygons.
280	7-8	Geometry	25	Planar and spatial shapes' constructions, transformations, properties, and classification	78_25_19	Study 3D shapes. Learn to calculate the areas and volumes of spheres, cylinders, and cones. Create nets of various geometric objects and understand their properties and relationships.
281	7-8	Analysis	23	Functional relationships	78_23_01	Quantitative relationships in everyday life situations and the kinds of functions that model them
282	7-8	Analysis	23	Functional relationships	78_23_02	Selection of the appropriate representation for the same quantity
283	7-8	Analysis	23	Functional relationships	78_23_03	Linear and quadratic relationships: identification and comparison
284	7-8	Analysis	23	Functional relationships	78_23_04	Describe dependencies both graphically and algebraically (direct and indirect proportionality)
285	7-8	Analysis	23	Functional relationships	78_23_05	Interpret graphs (increase and decrease of a function, slope and constant term, zeros of function)
286	7-8	Analysis	23	Functional relationships	78_23_06	Maps the elements of two concrete sets to each other
287	7-8	Analysis	23	Functional relationships	78_23_07	Illustrates the data in a data table graphically
288	7-8	Measuring	26	Measurements and Units	78_26_01	Estimation of measurement
289	7-8	Measuring	26	Measurements and Units	78_26_02	Measurable attributes of physical and mathematical objects
290	7-8	Measuring	26	Measurements and Units	78_26_03	Skills in units of measurement and their conversions

291	7-8	Measuring	26	Measurements and Units	78_26_04	Surface and volume calculation
292	7-8	Algebra	17	Natural numbers, Integers, and their operations	78_17_01	Varied systematic counting strategies in everyday life
293	7-8	Algebra	17	Natural numbers, Integers, and their operations	78_17_02	Adaptation of counting to the size of numbers
294	7-8	Algebra	17	Natural numbers, Integers, and their operations	78_17_03	Exact value, approximate value, and rounding
295	7-8	Algebra	17	Natural numbers, Integers, and their operations	78_17_04	Operations with negative numbers
296	7-8	Algebra	20	Rational numbers, fractions, and their operations	78_20_01	Arithmetic of fractions
297	7-8	Algebra	20	Rational numbers, fractions, and their operations	78_20_02	Opposite number, reciprocal value, absolute value
298	7-8	Algebra	20	Rational numbers, fractions, and their operations	78_20_03	operations with decimal numbers
299	7-8	Algebra	22	Problem-solving with equations, proofing	78_22_01	Problem-solving with equations (approximation, decomposition, or transposition method)
300	7-8	Algebra	22	Problem-solving with	78_22_02	Express linear and quadratic relationships with symbolic algebra

				equations, proofing		
301	7-8	Algebra	22	Problem-solving with equations, proofing	78_22_03	Modelling of everyday situations using mathematical representations and algebraic language
302	7-8	Algebra	22	Problem-solving with equations, proofing	78_22_04	Equivalence of algebraic expressions (linear and quadratic)
303	7-8	Algebra	22	Problem-solving with equations, proofing	78_22_05	Form and solve first-order equations and incomplete quadratic equations
304	7-8	Algebra	22	Problem-solving with equations, proofing	78_22_06	Operations of polynomials
305	7-8	Algebra	22	Problem-solving with equations, proofing	78_22_07	Forming and simplifying expressions
306	7-8	Algebra	22	Problem-solving with equations, proofing	78_22_08	Select solutions to inequalities that satisfy certain conditions
307	7-8	Algebra	22	Problem-solving with equations, proofing	78_22_09	Strategies for solving mathematical problems and evaluation of chosen strategies and methods
308	7-8	Algebra	22	Problem-solving with equations, proofing	78_22_10	Variables and their use in algebraic expressions, formulae, equations, and functions
309	7-8	Algebra	22	Problem-solving with equations, proofing	78_22_11	Decompose a problem into simpler parts, facilitating its computational interpretation
310	7-8	Algebra	24	Series (Sequences)	78_24_01	Numerical patterns and regularities
311	7-8	Algebra	24	Series (Sequences)	78_24_02	Form number sequences

312	7-8	Algebra	24	Series (Sequences)	78_24_03	Constructing, describing, and expressing patterns in number sequences and geometrical patterns
313	7-8	Algebra	30	Number theory, LCM, GCD, power, root	78_30_01	Divisibility of numbers
314	7-8	Algebra	30	Number theory, LCM, GCD, power, root	78_30_02	Divide numbers into prime factors
315	7-8	Algebra	30	Number theory, LCM, GCD, power, root	78_30_03	Calculates the lowest common denominator and the greatest common divisor
316	7-8	Algebra	30	Number theory, LCM, GCD, power, root	78_30_04	Power calculations with an integer exponent
317	7-8	Algebra	30	Number theory, LCM, GCD, power, root	78_30_05	Simplifying power expressions
318	7-8	Algebra	30	Number theory, LCM, GCD, power, root	78_30_06	The square root of square numbers
319	7-8	Algebra	31	Pattern usage/recognition	78_31_01	Recognition of patterns facilitates their computational interpretation
320	7-8	Mathematics, logic/set theory	18	Sets	78_18_01	Sorts elements into sets based on multiple criteria
321	7-8	Mathematics, logic/set theory	18	Sets	78_18_02	Subset recognition and illustration
322	7-8	Mathematics, logic/set theory	18	Sets	78_18_03	Numbers, sets of numbers illustration
323	7-8	Mathematics, logic/set theory	18	Sets	78_18_04	Set operations (complement, intersection, union)

324	7-8	Mathematics, logic/set theory	18	Sets	78_18_05	Set of rational numbers, infinite non-periodic decimal fractions
325	7-8	Mathematics, logic/set theory	18	Sets	78_18_06	Set of real numbers, properties, and usage
326	7-8	Mathematics, logic/set theory	19	Math and logic in everyday life	78_19_01	Interpreting and producing mathematical text
327	7-8	Mathematics, logic/set theory	19	Math and logic in everyday life	78_19_02	Reasoning and justification
328	7-8	Mathematics, logic/set theory	19	Math and logic in everyday life	78_19_03	Looking for rules and dependencies and presenting them precisely
329	7-8	Mathematics, logic/set theory	19	Math and logic in everyday life	78_19_04	Consider and determine the number of options (in math problems)
330	7-8	Mathematics, logic/set theory	29	Mathematical language, reasoning, logic, and combinatorics	78_29_01	Using appropriate mathematical language
331	7-8	Mathematics, logic/set theory	29	Mathematical language, reasoning, logic, and combinatorics	78_29_02	Describing, explaining, and justifying reasoning, procedures, and conclusions
332	7-8	Mathematics, logic/set theory	29	Mathematical language, reasoning, logic, and combinatorics	78_29_03	Strategies for deducing reasonable conclusions from a mathematical model
333	7-8	Mathematics, logic/set theory	29	Mathematical language, reasoning, logic, and combinatorics	78_29_04	Deduces truth values for propositions

334	7-8	Mathematics, logic/set theory	29	Mathematical language, reasoning, logic, and combinatorics	78_29_05	Expresses true and false statements
335	7-8	Mathematics, logic/set theory	29	Mathematical language, reasoning, logic, and combinatorics	78_29_06	Methods for overviewing all of the cases (in calculations)
336	7-8	Mathematics, logic/set theory	29	Mathematical language, reasoning, logic, and combinatorics	78_29_07	Illustrates tasks in graphs (Graph theory, Seven Bridges of Konigsberg)
337	7-8	Mathematics, logic/set theory	29	Mathematical language, reasoning, logic, and combinatorics	78_29_08	Usage of combinatorial principles
338	7-8	Probability and Statistics	21	Ratios, percentages	78_21_01	Understanding and representation of quantitative relationships
339	7-8	Probability and Statistics	21	Ratios, percentages	78_21_02	Comparison of decimals and percentages
340	7-8	Probability and Statistics	21	Ratios, percentages	78_21_03	Understanding of the concept of percent
341	7-8	Probability and Statistics	21	Ratios, percentages	78_21_04	Calculate the amount indicated by the percentage of the whole
342	7-8	Probability and Statistics	21	Ratios, percentages	78_21_05	Solve economic, finance, and everyday life problems related to percentages
343	7-8	Probability and Statistics	21	Ratios, percentages	78_21_06	Exchanges units of measurement regarding time, mass, length, area, and volume based on decimal thinking

344	7-8	Probability and Statistics	21	Ratios, percentages	78_21_07	Problem-solving by a linear and inverse relationship
345	7-8	Probability and Statistics	21	Ratios, percentages	78_21_08	Identify the multiplicities given the ratios
346	7-8	Probability and Statistics	27	Descriptive statistics	78_27_01	Relevant data to answer questions posed in statistical investigations
347	7-8	Probability and Statistics	27	Descriptive statistics	78_27_02	Strategies for drawing conclusions from a sample in order to make judgments and appropriate decisions
348	7-8	Probability and Statistics	27	Descriptive statistics	78_27_03	Strategies for collecting and organizing data for a single variable
349	7-8	Probability and Statistics	27	Descriptive statistics	78_27_04	Analysis and interpretation of statistical tables and graphs of qualitative, discrete quantitative, and continuous quantitative variables
350	7-8	Probability and Statistics	27	Descriptive statistics	78_27_05	Understanding of the average value
351	7-8	Probability and Statistics	27	Descriptive statistics	78_27_06	Determining frequency, relative frequency, and median
352	7-8	Probability and Statistics	27	Descriptive statistics	78_27_07	Problem-solving with proportion
353	7-8	Probability and Statistics	27	Descriptive statistics	78_27_08	Interprets data in tables, selects the appropriate visualization method and creates the visualization
354	7-8	Probability and Statistics	27	Descriptive statistics	78_27_09	Calculates the average of a data series, determines the most common value (mode), the middle datapoint (median), and compares these
355	7-8	Probability and Statistics	28	Probability theory	78_28_01	Identification of deterministic and random phenomena
356	7-8	Probability and Statistics	28	Probability theory	78_28_02	Simple experiments: planning, performance, and analysis of the associated uncertainty

357	7-8	Probability and Statistics	28	Probability theory	78_28_03	Assignment of probabilities to experiments
358	7-8	Probability and Statistics	28	Probability theory	78_28_04	Calculate the probabilities
359	7-8	Probability and Statistics	28	Probability theory	78_28_05	Calculate the standard deviation
360	7-8	Probability and Statistics	28	Probability theory	78_28_06	Explain impossible, certain, less/more likely statements

Learning Paths

Based on the connections and analysis, the learning paths for the given age groups are defined (Tables 2-4).

Table 2. Learning paths for age groups 3 and 4

Grade	Math Area	Topic ID and Category	
Grade	**Math Area**	**No**	**Topic categories**
3-4	Geometry	9	Shapes and objects, and their properties
3-4	Geometry	11	Orientation in space and on a plane
3-4	Geometry	25	Planar and spatial shapes' constructions, transformations, properties, and classification
3-4	Analysis	10	Constructions and Translations
3-4	Measuring	6	Measurements and measurement tools
3-4	Measuring	26	Measurements and Units
3-4	Algebra	2	Problem solving
3-4	Algebra	3	Comparison, sorting
3-4	Algebra	4	Counting, approximations
3-4	Algebra	7	Equations, operations
3-4	Algebra	8	Mental calculations
3-4	Algebra	12	Relationships
3-4	Algebra	15	Columnar operations
3-4	Algebra	16	Part-Part-Whole Relationships
3-4	Algebra	17	Natural numbers, Integers, and their operations
3-4	Algebra	31	Pattern usage/recognition
3-4	Number system	5	Digits, numbers, number systems
3-4	Mathematics, logic/set theory	1	Categorization, Classification
3-4	Mathematics, logic/set theory	19	Math and logic in everyday life
3-4	Probability and Statistics	13	Data collection and management
3-4	Probability and Statistics	14	Randomness
3-4	Probability and Statistics	21	Ratios, percentages

Table 3. Learning paths for age groups 5 and 6

Grade	Math Area	Topic ID and Category	
Grade	**Math Area**	**No**	**Topic categories**
5-6	Geometry	9	Shapes and objects, and their properties
5-6	Geometry	11	Orientation in space and on a plane
5-6	Geometry	25	Planar and spatial shapes' constructions, transformations, properties, and classification
5-6	Analysis	10	Constructions and Translations
5-6	Analysis	23	Functional relationships
5-6	Measuring	6	Measurements and measurement tools
5-6	Measuring	26	Measurements and Units
5-6	Algebra	3	Comparison, sorting
5-6	Algebra	17	Natural numbers, Integers, and their operations
5-6	Algebra	20	Rational numbers, fractions, and their operations
5-6	Algebra	22	Problem-solving with equations, proofing
5-6	Algebra	24	Series (Sequences)
5-6	Algebra	30	Number theory, LCM, GCD, power, root
5-6	Algebra	31	Pattern usage/recognition
5-6	Number system	5	Digits, numbers, number systems
5-6	Mathematics, logic/set theory	18	Sets
5-6	Mathematics, logic/set theory	19	Math and logic in everyday life
5-6	Probability and Statistics	13	Data collection and management
5-6	Probability and Statistics	14	Randomness
5-6	Probability and Statistics	21	Ratios, percentages
5-6	Probability and Statistics	27	Descriptive statistics
5-6	Probability and Statistics	28	Probability theory

Table 4. Learning paths for age groups 7 and 8

Grade	Math Area	Topic ID and Category	
Grade	**Math Area**	**No**	**Topic categories**
7-8	Geometry	25	Planar and spatial shapes' constructions, transformations, properties, and classification
7-8	Analysis	23	Functional relationships
7-8	Measuring	26	Measurements and Units
7-8	Algebra	17	Natural numbers, Integers, and their operations
7-8	Algebra	20	Rational numbers, fractions, and their operations
7-8	Algebra	22	Problem-solving with equations, proofing
7-8	Algebra	24	Series (Sequences)
7-8	Algebra	30	Number theory, LCM, GCD, power, root
7-8	Algebra	31	Pattern usage/recognition
7-8	Mathematics, logic/set theory	18	Sets
7-8	Mathematics, logic/set theory	19	Math and logic in everyday life
7-8	Mathematics, logic/set theory	29	Mathematical language, reasoning, logic, and combinatorics
7-8	Probability and Statistics	21	Ratios, percentages
7-8	Probability and Statistics	27	Descriptive statistics
7-8	Probability and Statistics	28	Probability theory