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## COMATH guidelines for teachers

The results are based on the work within the project “Computational Thinking and Mathematical Problem Solving, an Analytics Based Learning Environment” (CT&MathABLE). Coordination: Prof. Valentina Dagienė, Vilnius University (Lithuania). Partners: Ankara University (Türkiye), Eötvös Loránd University (Hungary), Gedminų Progymnasium (Lithuania), KTH Royal Institute of Technology (Sweden), Mamak Özkent Akbilek Middle School (Türkiye), University of Basque Country (Spain), University of Turku (Finland). The project has received co-funding by the European Union under the Erasmus+ Programme KA220.

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## General overview

The COMATH Teacher Guidelines help teachers, school administrators, and researchers administer and interpret the COMATH assessment, which measures Computational Thinking (CT) and Algebraic Thinking (AT) skills in students aged 9–14. There are three levels: COMATH 1, 2, and 3 for children aged 9–10, 11–12, and 13–14, respectively.

The assessment instrument COMATH was developed by the Turku Research Institute for Learning Analytics, University of Turku, in collaboration with the project partners. COMATH draws on educational theories and curriculum standards from six countries and was reviewed by researchers, teacher educators, teachers, and students. Its validity has been confirmed through large-scale pilot studies involving more than 6,400 students and over 200 teachers.





## Target groups

Teachers, school administrators, and researchers

### Keywords

Computational Thinking; Algebraic Thinking; assessment instrument; K–9 education

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## 1. About These Guidelines

### 1.1 Purpose

These guidelines support teachers and researchers in using the COMATH assessment instrument to identify 9–14-year-old students' Computational Thinking (CT) and Algebraic Thinking (AT) skills. The results can help guide teaching strategies and student support.

### 1.2 What's Inside

1. What is COMATH?
2. How to use COMATH
3. Understanding the results
4. References
5. Appendices with test items, answers, and scoring

## 2. What are CT and AT?

### 2.1 What is Computational Thinking (CT)?

CT is the ability to solve problems effectively using logical and structured thinking, drawing on concepts from computer science. Key components include:

- **Decomposition:** Breaking problems into smaller parts
- **Abstraction:** Focusing on what matters and ignoring irrelevant detail
- **Data Collection, Analysis & Representation:** Gathering and interpreting data to identify relationships
- **Algorithmic Thinking:** step-by-step solutions
- **Evaluation & Adjustment:** Testing and improving solutions
- **Transferability:** Applying solutions in different situations

### 2.2 What is Algebraic Thinking (AT)?

AT involves recognising and expressing general mathematical relationships using different forms of representation. It includes:

- Generalising patterns and functions
- Understanding equality and equations
- Reasoning logically
- Using variables and representations.

## 3. What is COMATH?

### 3.1 Overview

COMATH (CO = Computational; MATH = Mathematical) is a CT and AT assessment instrument created by the Turku Research Institute for Learning Analytics, University of Turku, in collaboration with international partners through the CT&MathABLE project (Erasmus+ Grant Agreement No. 2022-1-LT01-KA220-SCH-000088736).

The instrument is grounded in educational theories and curriculum standards from six countries—**Finland, Hungary, Lithuania, Spain, Sweden, and Türkiye**—and was reviewed by researchers, teacher educators, and teachers. It was validated through two pilot studies with over **6,400 students and 200 teachers**.

### 3.2 Key Features

COMATH is:

- **Curriculum-neutral:** can be adapted to national/local standards
- **Flexible:** can be used at any point during the school year
- **Targeted:** Suitable for students aged 9–14, with level-specific tests:
  - **COMATH 1:** Ages 9–10
  - **COMATH 2:** Ages 11–12
  - **COMATH 3:** Ages 13–14

Each level includes:

- A **CT test** focused on algorithmic thinking and other CT skills
  - COMATH 1-CT: 12 items
  - COMATH 2-CT: 12 items
  - COMATH 3-CT: 14 items
- An **AT test** covering six core areas:
  1. **Generalised Arithmetic:** Understanding mathematical relationships, including properties of numbers and operations
  2. **Equivalence & Equations:** Understanding equal relationships
  3. **Functional Thinking:** Identifying and generalising patterns
  4. **Representation:** Using tables, diagrams, symbols, etc.
  5. **Transformation:** Manipulating algebraic expressions, including simplifying and expanding them
  6. **Transversal Skills:** Reasoning, generalising, modelling, and validating

Each AT test is ordered from easier to more difficult and rotates through the six core areas in sequence (e.g., item 1 from Area 1, item 2 from Area 2, etc., then repeats). This ensures students engage with all areas evenly.

### 3.3 Test Structure

**Table 1.** Number of AT items by age group

Core Area	COMATH 1 (9–10 y)	COMATH 2 (11–12 y)	COMATH 3 (13–14 y)
1. Generalised Arithmetic	17	18	19
2. Equivalence & Equations	13	13	14
3. Functional Thinking	7	7	5
4. Representation	3	4	7
5. Transformation	5	5	8
6. Transversal Skills	6	7	6
<b>TOTAL</b>	<b>51</b>	<b>54</b>	<b>59</b>

### 3.4 Access and Licensing

COMATH is available under a Creative Commons Attribution-ShareAlike 4.0 International License ([CC BY-SA 4.0](https://creativecommons.org/licenses/by-sa/4.0/)). You are free to use, adapt, and share it—just credit the original source, indicate any changes, and use the same licence for your version.

Available languages: English, Finnish, Hungarian, Lithuanian, Turkish, Swedish, Spanish, Basque, and Catalan.

You can download the instrument from the project website after completing a registration form. The form collects data only for statistical purposes and resource improvement.

## 4. How to Use COMATH

### 4.1 Getting Ready

- Review CT and AT concepts
- Read through the test and guidelines
- Get parental consent if required (e.g., for research use)

### 4.2 Administering the Test

- Use two lessons (40–45 minutes each): one for CT, one for AT
- CT and AT tests can be given in any order
- Allow time for explaining instructions
- Present the test as a way to explore students' thinking, not for grading
- Encourage everyone to participate
- Avoid helping or giving hints
- Offer support for students with additional needs (e.g., extra time, quiet space)

### 4.3 Materials Needed

Prepare the following:

- Printed test and optional answer sheets
- Pens or pencils and erasers
- Scratch paper
- **Note:** Calculators are not allowed.

## 5. Understanding the Results

### 5.1 Interpreting Results

- Look at the class's lowest, average, and highest scores
- Identify common challenges or strengths
- Group students by similar needs
- Reflect on whether difficulties are concept- or test-taking-related

### 5.2 Using Results to Support Learning

- Use results to inform teaching strategies
- Share findings with students and parents positively, emphasise progress
- Combine COMATH results with other classroom observations for a complete view

## 6. References

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## 7. Appendices

[Appendix A](#): COMATH 1-CT Test Items, Answer Keys, and Scoring Guide

[Appendix B](#): COMATH 1-AT Test Items, Answer Keys, and Scoring Guide

[Appendix C](#): COMATH 2-CT Test Items, Answer Keys, and Scoring Guide

[Appendix D](#): COMATH 2-AT Test Items, Answer Keys, and Scoring Guide

[Appendix E](#): COMATH 3-CT Test Items, Answer Keys, and Scoring Guide

[Appendix F](#): COMATH 3-AT Test Items, Answer Keys, and Scoring Guide